discrete2continuous.stanford.edu

A website supplementary to:

The Black–Scholes–Merton Model as an Idealization of Discrete-Time Economies

Further readings and references

This document, which will be revised from time to time, provides an annotated list of further readings and references, with links to the references when available. (October 2019: This is the first "edition of this note" and is just a beginning. I expect to have a significantly more comprehensive list of papers and links in due course.)

1. In the monograph, Proposition 5.1*b*, which is central to developments in the book, is not proved. Instead, reference is made to Kreps and Schachermayer, "Asymptotic Synthesis of contingent claims in a sequence of discrete-time markets," in which the proof is provided. This paper is available online at SSRN:

   https://ssrn.com/Abstract=3402645

2. In the monograph, I conjecture that, for a general random-walk model, in the sense of Chapter 5, if the basic random variable ζ has bounded support, then the solutions of expected-utility-maximization problems in the discrete-time economies give, in the limit, the same level of expected utility as a consumer can attain in the continuous-time limit, BSM economy. I prove, more or less, that the consumer can do no worse asymptotically in the discrete-time economies than in the limit (this is, again more-or-less, the content of Proposition 5.2 in the monograph). But I do not exclude the possibility that she can do strictly better, asymptotically.

   Since finishing the monograph, I worked further on this question with Walter Schachermayer, and we were able to obtain some very nice results concerning this conjecture. We work (only) in the context of utility functions U that are strictly increasing, strictly concave, differentiable, and have $\lim_{x\to 0} U'(x) = \infty$ and $\lim_{x\to\infty} U'(x) = 0$. Within the context of such utility functions, we confirm

the conjecture if

$$\limsup_{x\to\infty} xU'(x)/U(x) < 1,$$

and we provide a counterexample if this lim sup = 1. (This lim sup is called by Kramkov and Schachermayer, 1999, the asymptotic elasticity of the utility function U.)

These results are given in Kreps and Schachermayer, "Convergence of Optimal Exptected Uility for a Sequence of Discrete-Time Markets," which is posted at

https://.ssrn.com/Abstract=3417898

3. Readers of the paper "Convergence of Optimal Expected Utility..." will benefit by first reading Kramkov and Schachermayer (1999), "The asymptotic elasticity of utility functions and optimal investment in incomplete markets," *The Annals of Applied Probability,* Volume 9, 904-50. A pdf of the paper can be downloaded at

https://projecteuclid.org/euclid.aoap/1029962818