

Appendix 4. Expected Utility as a Normative Decision Aid

This appendix to Chapter 18 discusses the expected utility model as a *normative* model or decision aid. The protagonist is not some abstract decision maker; instead, this appendix focuses on *you*.

- First we justify the expected utility model by presenting five qualitative axioms for behavior that, if you subscribe to them, imply you should make decisions under uncertainty by maximizing *your* expected utility.
- Then we discuss how to assess your own (subjective) utility function.
- We discuss some more-advanced procedures you can use, at least in some instances, to improve your assessment of your own utility function.
- We conclude by discussing three reasons why you might *not* want to use the expected utility model.

The purpose of this appendix is to help *you* make better choices when you face uncertainty. Whether in your personal or business life, you will often face a choice among actions that have uncertain consequences, and you are likely to find that the choice is a hard one. A simple example illustrates how hard such a choice can be.

Imagine that I offer you the (personal) choice of the five gambles depicted in Figure A4.1. You can have only one of these. Which do you choose? How confident are you that your choice is the right one for you?

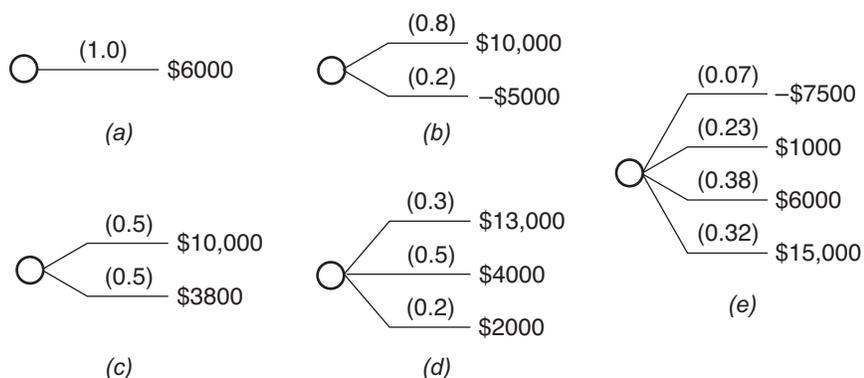


Figure A4.1. Five gambles. Given a choice of one of these five gambles, which would you select?

If you are like most people, this is not an easy decision. It is hard because it requires that you integrate in your mind all aspects of a gamble with several prizes and probabilities such as 0.07 and 0.38.

Suppose, though, I could convince you that you would like your choice concerning such gambles to conform to expected-utility maximization. Further suppose I could direct you to an organization that, using an EEG (electroencephalogram, a device that measures brain-wave patterns), can plot your personal utility function. Then such decisions would become a simple matter of mechanically computing your expected utility—and, if you want, your certainty equivalent—for each gamble, choosing whichever gamble gives you the largest expected utility or CE.

While I cannot deliver on this two-barreled promise, I can come close: I hope to convince you that, subject to some important caveats, your choice in this sort of situation should conform to the conscious maximization of expected utility. And in place of the promised application of electroencephalography, I can show you how, making relatively simple subjective judgments, you can get a good approximation to your utility function to use in problems like this one.

A4.1. Justifying the Expected Utility Procedure

The first step is to convince you that you want your choice to conform to the expected-utility model. For the time being, I do this for gambles with objective probabilities. In my illustrative examples, the prizes always are monetary, but this is for expositional convenience only; what I say applies to gambles with objective probabilities and any sort of prizes.

I also throw into the set of possible objects of choice so-called compound lotteries or gambles. These are objects in which a sequence of random events may be conducted before you get a prize. For instance, I might roll a fair die. If the die comes up one or two spots (probability $1/3$), I give you \$500. If it comes up three spots (probability $1/6$), I flip a fair coin: Heads, you get \$400; tails, I flip again; and if heads this time, you get \$500 and \$0 if tails. If the original throw of the die comes up four, five, or six spots (probability $1/6$ each), you pay me \$10 times the number of spots. This compound lottery is depicted in Figure A4.2(a). (Ignore Figure A4.2(b) for now.)

I will ask you to make pairwise comparisons between gambles. Specifically, given any pair of gambles, I ask you to say whether you consider the first to be *as good as* the second and whether you think the second is *as good as* the first. About these preference judgments,

- I do not preclude the possibility that you say both things about a par-

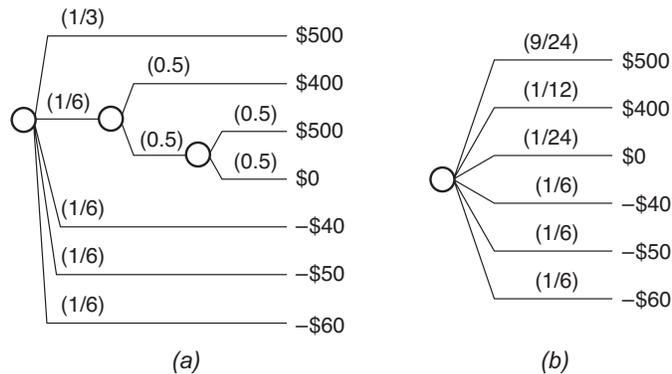


Figure A4.2. A compound gamble and the one-stage gamble to which it reduces. Panel a shows a compound gamble, a gamble that is conducted in several steps. This compound gamble reduced to the one-stage gamble in panel b by the rules of probability, so if you obey property 1 (reduction of compound gambles), you are indifferent between them.

ticular pair of gambles, in which case I will interpret your statements as saying that you are *indifferent* between them.

- I do not preclude the possibility, at least not yet, that you are unwilling to express a judgment either way.
- When you say that gamble A is as good as B, but you do not say that B is as good as A, I interpret this as you *strictly prefer* A to B.

Finally, I assume that you would like to connect your preference judgments and your choices as follows: If you must choose one gamble from a set of gambles, you would like to choose one that you judge to be as good as everything else in the set. (If more than one meets the “as good as everything else” test, you are happy with any that do so.)

What more can we say about your preferences? Here is a list of five properties that your preferences over gambles may or may not satisfy. As you read these properties, ask yourself whether they seem normatively desirable; that is, do you want your preferences to satisfy them?

Property 1. Reduction of compound gambles. You are indifferent between any compound gamble and the simple (one-stage) gamble to which it reduces using the rules of probability theory.

If your preferences satisfy this property, you are indifferent between the gambles in panels a and b of Figure A4.2, since if we multiply out branches and then add the probabilities of outcomes giving the same dollar prize in panel a, we get the gamble in panel b.

Property 2. Completeness. For any two gambles A and B, you are willing to

say that either A is as good as B , or that B is as good as A , and possibly (if you are indifferent between them) both.

Do not agree to this property too quickly: For most people, expressing preference judgments is difficult in some cases. But the question here is whether you *want* to be able to express a judgment for every pair of gambles and not whether you *can*.

Property 3. Transitivity. *If gamble A is as good as gamble B in your opinion, and B is as good as C , then you judge that A is as good for you as is C .*

I assume you have no problem with transitivity as a desirable property for your preferences.

Property 4. Continuity. *Suppose you feel that gamble A is strictly better than gamble B and that B in turn is strictly better than C . Construct two compound gambles: Let gamble D be a gamble in which you get the gamble A if 10,000 coins flipped in a row come up heads every time and you get the gamble C otherwise. Let gamble E be a gamble in which you get gamble C if 10,000 coins flipped in a row come up tails every time, and you get the gamble A otherwise. Then gamble E is strictly better than gamble B , which in turn is strictly better than gamble D . Or, if this is not true, some finite number larger than 10,000 makes it true.*

What a mouthful. But the idea is simple. Gamble D is “almost” the same as gamble C , since it is gamble C with probability $1 - 0.5^{10000}$. Since you believe gamble B is strictly better than gamble C , gamble B ought to be strictly better for you than gamble D , the “almost gamble C ” gamble. If it is not, then we can find some number bigger than 10,000 that will make it so. That is half of property 4; the other half does the same thing on the other side.

Property 5. Substitution. *Suppose you strictly prefer gamble A to gamble B . Take any third gamble C and any probability $p > 0$, and construct the following two compound gambles. Gamble D gives you gamble A with probability p and gamble C with probability $1 - p$. Gamble E gives you gamble B with probability p and gamble C probability $1 - p$. That is, the difference between the two compound gambles is that with gamble D , you get gamble A with probability p , while in E , you get gamble B with this probability; in both D and E , you get C with probability $1 - p$. Since you strictly prefer the A to B and since $p > 0$, it must be (and this property says that it is) that you prefer D to E .*

Figure A4.3 provides a diagrammatic representation of Properties 4 and 5. Do not worry about the names I give these properties and do not waste your time memorizing either the names or the properties themselves. Instead, concentrate on whether you find them to be *normatively desirable* rules for your preferences to satisfy. Would you like to make choices based on

preferences that always satisfy these properties? If not, which one(s) would you be willing to violate?

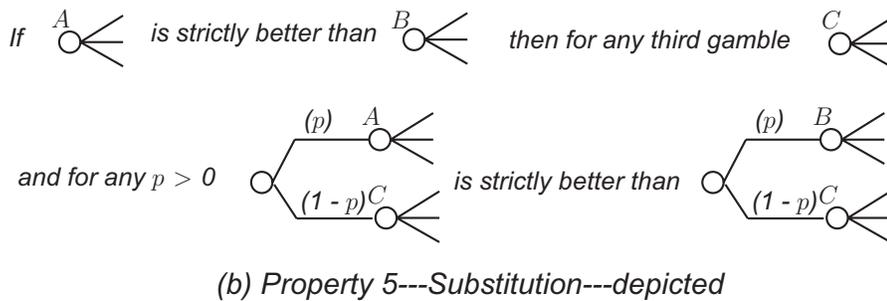
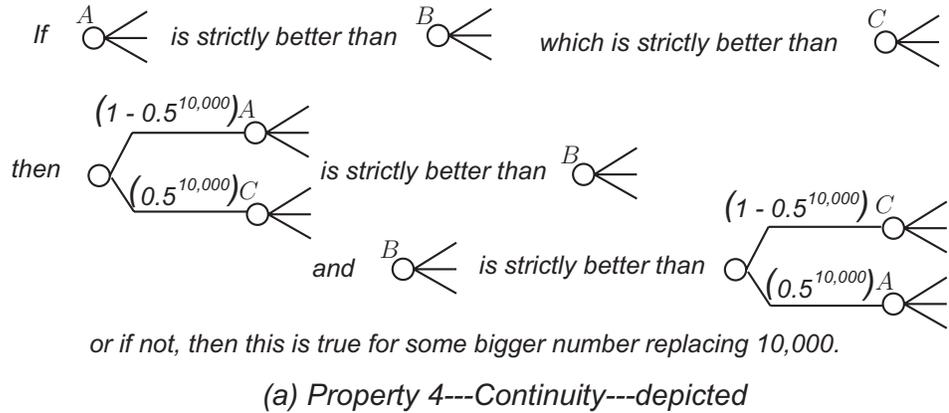


Figure A.4.3. Properties 4 and 5 in pictures

I hope you think these are normatively desirable rules; you would like the choices you make to be based on preferences that satisfy these properties. Most folks, thinking about choosing among gambles, find these properties entirely reasonable, in most situations. (But do not be too fast to agree to them. Wait at least until the end of this appendix.)

In the hope that you decide that these five properties are normatively desirable, I can give the punchline.

Mathematical fact. Any set of preferences among gambles that conforms to the preceding five properties is consistent with the expected-utility model in the sense that one gamble is preferred to another if and only if the first gives higher expected utility, for some utility function u defined on the set of prizes. And any set of preferences that corresponds to maximizing expected utility for some utility function u obeys the five properties.

Therefore, if you find these five properties to be normatively desirable, you

want to choose based on expected utility maximization for some utility function u . Please note carefully that in Chapter 18 of the text, expected utility is advanced as an as-if theory. We do not see people computing expected utilities and letting those computations guide their choices; expected utility is a descriptive model to the extent that people choose *as if* they were doing this. But now, in a normative vein, there is nothing as-if about this. I do not suggest the normative desirability of choosing *as if* you were computing expected utilities. I am advocating—and if you find the five properties are desirable in terms of good decision making, you are agreeing to—the *explicit* use of expected-utility calculations to improve on your otherwise fallible decision making under uncertainty.

Why is this alleged mathematical fact a fact? That takes a fair bit of mathematics, but if you are curious, consult Chapter 5 of Kreps, *Microeconomic Foundations I: Choice and Competitive Markets*, or any similar book on choice theory.

It remains to assess your personal utility function. Actually, a few things remain to say about the five properties and this basic conclusion, but it is positionally easier to say those things after discussing how to assess your utility function.

A4.2. Assessing Your Utility Function

If there were some way to have a machine discover your personal utility function u , choice among gambles would never be a problem for you, at least as long as you subscribe to the five properties. Unhappily, scientists have yet to invent a machine capable of finding your personal utility function. *You* must do it. But there are ways to make this procedure *relatively* easy.

Suppose you must choose among a number of gambles. To be very concrete, suppose you had to choose among the gambles in Figure A4.1. First note that the range of prizes here runs from a loss of \$7500 to a gain of \$15,000. So we obtain your utility function for that range of prizes.

Arbitrarily set the utility of $-\$7500$ to equal 0 and the utility of \$15,000 to equal 1. As discussed in Chapter 18 in the text, utility functions (for expected utility) can be translated and stretched or shrunk to fit any scale you wish; in practical terms, this means you can set the utility of any two dollar values as you like. It will be convenient to assign the worst possible prize a utility of 0 and the best possible prize a utility of 1, so we do that.

Next ask yourself three questions:

1. *What amount of money obtained for sure would be just as good as a gamble in which you receive \$15,000 with probability 1/2 and you lose \$7500 with*

probability 1/2?

You may object that this is a hard question to answer precisely. How can you tell precisely what is your certainty equivalent—that is what we ask you for—for the 50–50 gamble with prizes \$15,000 and $-\$7500$? The simple answer is that you cannot tell this precisely. But it is probably a lot easier for you to make this judgment than to choose among the five gambles in Figure A4.1. So suppose, for the sake of argument, that the answer you come up with is \$2000. That is, if I offered you either the risky gamble or \$2250, you would take \$2250 for certain in preference to the gamble, but you prefer the gamble to \$1750. Then we can continue,

2. *What amount of money obtained for sure would be just as good for you as a gamble in which you receive \$15,000 with probability 1/2 or you receive **\$2000** with probability one-half?*

Suppose for the sake of argument that the value is \$7500. (Why is the figure \$2000 emphasized? I'll explain in a bit.)

3. *What amount of money obtained for sure would be just as good for you as a gamble in which you lose \$7500 with probability 1/2 or you receive **\$2000** with probability one-half?*

Since this gamble is probably worse than not gambling at all, I may have to amend the question to read:

- 3'. *How much would you be willing to pay, to get out of having to take a gamble where you lose \$7500 with probability 1/2 or gain **\$2000** with probability 1/2?*

If the answer to question 3 is $-\$X$, the answer to question 3' is $\$X$. Suppose that $\$X$ is \$3000. That is, you would be willing to pay \$2750 rather than take the gamble, but you would rather take the gamble than pay \$3250.

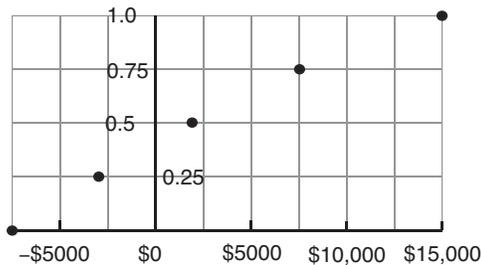
The precise terms of questions 2 and 3 (or 3') depend on your answer to question 1. If your answer to question 1 is \$1500, then question 2 would concern a gamble with prizes \$15,000 and \$1500 instead of \$15,000 and \$2000. This explains the bold-faced italic: Your answer to question 1 is substituted in these two places.

Given answers to these three questions, we can begin to build your personal utility function u . We set the scale so that $u(15,000) = 1$ and $u(-7500) = 0$. Therefore, the first answer you gave establishes that $u(2000) = 0.5$ on this scale. Why? Because a gamble where you get prizes \$15,000 and $-\$7500$, each with probability $1/2$, has expected utility 0.5 on the scale we set. If this gamble is indifferent to \$2000 for sure—and you said it was, or so we suppose, for sake of illustration—then $u(2000)$ must equal 0.5.

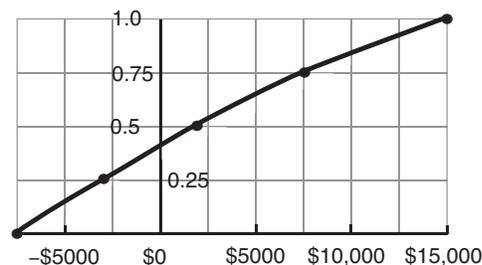
Similarly, your second answer establishes that $u(7500) = 0.75$ on this

scale, and your third answer establishes that $u(-3000) = 0.25$. Why? Because if $u(2000) = 0.5$, then a gamble with equally likely prizes \$15,000 and \$2000 has expected utility $(0.5)[1.0] + (0.5)[0.5] = [0.75]$, and you said you were indifferent between this gamble and \$7500 for sure. Similarly, the gamble that gives you $-\$7500$ or \$2000, each with probability $1/2$, has the expected utility 0.25, and you are indifferent between this gamble and a sure loss of \$3000.

So we can plot five points of your utility function, as in Figure A4.4(a). And then we can rough in a utility function for you, as in Figure A4.4(b). This is a pretty rough rough-in, but if you use this function to choose among risky gambles, you will probably get a pretty good approximation to your true preferences.



(a) Five points on your utility function, obtained from answers to relatively simple questions



(b) Roughing in a utility function, using the five points

Figure A4.4. Plotting your utility function. By asking you to supply certainty equivalents for *relatively simple* gambles—with two equally-likely prizes—we can construct a fairly good approximation to your utility function.

The murmurs I heard after step 1 are growing in volume: You may be thinking that this is crazy. You have to make choices among risky gambles, and this procedure is supposed make those choices relatively easy. But the procedure requires very fine judgments from you about indifference among risky gambles and sure things. What is gained by that?

The gain comes from the fact the very fine judgments I ask you to make are the simplest judgments of this kind you can be asked to make; you are asked to compare a gamble having two equally likely prizes with a sure thing. A difficulty most people have in choosing among risky gambles is that their judgment is poor concerning gambles having lots of prizes and probabilities like 0.38 and 0.23. How likely is probability 0.23? If a gamble has four different prizes, with probabilities ranging from 0.38 to 0.07, how are the prizes to be combined and aggregated to get an overall sense of how good is the gamble? It may not be *easy* to compare a gamble having two equally likely prizes with a sure thing, but it is probably *easier* to do than to deal

directly with more complex gambles. You are probably going to come closer to a sense of your preferences with simple comparisons than more complex comparisons. The point is, if you find the five properties given earlier to be normatively reasonable, then you can take judgments you make in *relatively* simple situations and build from them your utility function u , which you can then use to evaluate more complex choices.

And make no mistake, the approximate utility function in Figure A4.4(b), applied mechanically to the five gambles in Figure A4.1, will give you a pretty good sense of your preference among the five, *if* the answers to the 50–50 gamble questions used to rough in this utility function reflect your subjective judgment. Of course, it is possible that you made an error in, say, the first judgment: Perhaps, your certainty equivalent for a 50–50 gamble with prizes \$15,000 and $-\$7500$ is closer to \$1500 than \$2000. But that error in judgment is more likely than not going to affect a comparison of the five gambles in Figure A3.1 somewhat uniformly. Moreover, any sensible use of a roughed-in utility function involves a computation of certainty equivalents, which gives you a sense of how much better (per your analysis) one gamble is than another. If the difference in CEs is small, then you cannot trust your analysis quite as much as if the difference is large. But, if the difference is small, you have less at stake if you follow the recommendations of your analysis and it is wrong.

Finally, we can do some relatively simple things to give you more confidence in the utility function you assess. Here are three of those things:

A Consistency Check

The first way to improve your assessment is a simple consistency check. Suppose you gave the answers listed previously: On a scale where $u(-7500) = 0$ and $u(15,000) = 1$, $0.5 = u(2000)$, and then $0.75 = u(7500)$ and $0.25 = u(-3000)$. Now ask, What is your certainty equivalent for a gamble with prizes \$7500 and $-\$3000$, each with probability $1/2$?

Because the expected utility of this gamble is $(0.5)[0.75] + (0.5)[0.25] = [0.5]$, the answer consistent with your other judgments is \$2000.

In most cases, people do not pass this consistency check the first time out. It takes some thinking and even a bit of fudging with the three subjective judgments until you are happy that the prize with utility 0.5 is the certainty equivalent of a gamble with equally likely prizes the utilities of which are 0.25 and 0.75. But this is a quick and easy way to check on your judgments.

Framing: Defeating the Zero Illusion

Although we did not say so explicitly, in our assessment of your utility function, the units on the horizontal axis were your net winnings or losses

from this particular gamble. As noted in the text, individuals are prone to a zero illusion when presented with gambles framed in this fashion.

Hence, as a practical matter, when assessing your own utility function, you should attempt to defeat, or at least check on, the zero illusion by framing and reframing questions.

You can often eliminate the illusion altogether if you frame all gambles in terms of your net bank account after the gamble is conducted. For instance, suppose you are offered a choice of the five gambles in Figure A4.1 and your net (liquid) asset balance is \$24,220. Rather than thinking that the range of possible outcomes is $-\$7500$ to $\$15,000$, think in terms of a range for your possible net (liquid) asset balance between $\$16,720$ and $\$39,220$ or, probably better, a slightly larger range with even numbers for endpoints, such as from $\$15,000$ to $\$40,000$. Instead of beginning by trying to work through your certainty equivalent for a gamble with prizes $-\$7500$ to $\$15,000$, each with probability $1/2$, try to decide what asset balance *for sure* would be just as good to you as the situation where, based on a coin flip, your liquid asset balance will be either $\$15,000$ or $\$40,000$. Frame everything in terms of your net asset balance, and you are likely to come up with numbers that more accurately reflect how you really feel about things.

Using Constant Risk Aversion

A final technique for improving the quality of utility function you assess is very powerful *when it can be applied legitimately*. I illustrate using myself as guinea pig.

Given the current state of my bank account and my prospects for future income, I think it is reasonable to assume that I have roughly constant risk aversion for gambles with prizes ranging from $-\$50,000$ to $\$150,000$. (This was written for the first time in 1996. By the time you read this, it will still be valid, although the range over which I am comfortable with constant risk aversion will have grown and my level of risk aversion in that range will have fallen, both consequences of the fact that I've done moderately well financially.)

The italicized sentence is the key to everything that follows. It has the following explanation: I am risk averse and I believe my level of risk aversion decreases in my total wealth. That is, the risk premium I attach to some given gamble would likely decrease if someone were to give me $\$1$ million to keep. I am not quite sure what my attitude toward risk would be if my net asset position were close to $\$0$, but as long as my assets stay roughly where they are, I do not believe my attitude toward a given risk would change much. The key question is, What does *stay roughly where they are* mean? Given the money I have in the bank and in other assets, I think a loss of $\$50,000$ or a gain of $\$150,000$ would not materially affect my attitude toward risk. A

good test for this is, given recent gyrations in the stock market, I probably do not even know my net asset position to the nearest \$50,000 or so.

This means that, over the range of prizes $-\$50,000$ to $\$150,000$, I have, roughly, exponential utility. That is, my utility function for winnings and losses in this range has approximately the form $u(x) = -A e^{-\lambda x} + B$, where x is my net from current gambles, to be added to my assets, and $A > 0$ and B are constants. The values of the constants A and B are irrelevant if I want to evaluate expected utilities to choose among gambles, so I set them to be $A = 1$ and $B = 0$. Then, *finding my utility function over the range from a \$50,000 loss to a \$150,000 gain comes down to finding my coefficient of risk aversion, λ .*

Now what? I give three intuitively derived CEs of my own. I intentionally pick three 50–50 gambles with different ranges of prizes but all prizes within the band of $-\$50,000$ to $\$150,000$:

- For a gamble with prizes \$0 and \$150,000, each with probability 1/2, my CE is approximately \$60,000.
- For a gamble with prizes \$0 or \$10,000, each with probability 1/2, my CE is \$4500.
- For a gamble with prizes \$0 or \$50,000, each with probability 1/2, my CE is \$21,000.

Let me assure you that this is my best subjective judgment as to my CEs. I am not fudging these numbers.

Take my first judgment, that my CE for \$0 or \$150,000, each with probability 1/2, is \$60,000. Remember that I have constant risk aversion over this range, so my utility function has the form

$$u(x) = -e^{-\lambda x}.$$

Then the gamble has expected utility

$$(1/2)(-e^{-\lambda \cdot 0}) + (1/2)(-e^{-\lambda \cdot 150000}) = -0.5(1 + e^{-\lambda \cdot 150000}).$$

On the other hand, the (expected) utility of \$60,000 for sure is $-e^{-\lambda \cdot 60000}$. If my CE judgment is correct, λ must be set so that these two utility levels are equal, or

$$-0.5(1 + e^{-\lambda \cdot 150000}) = e^{-\lambda \cdot 60000}.$$

EXCEL can solve for λ here (I cannot do it analytically), giving $\lambda = 0.00000548$.

But my other two CE judgments are also sufficient to “determine” the value of λ . It should satisfy

$$-0.5(1 + e^{-\lambda \cdot 10000}) = -e^{-\lambda \cdot 4500} \quad \text{and} \quad -0.5(1 + e^{-\lambda \cdot 50000}) = -e^{-\lambda \cdot 21000}.$$

The first equation gives $\lambda = 0.0000402$, and the second gives $\lambda = 0.000013$. Because we get different values of λ , my intuitive subjective judgment is inconsistent with the expected-utility model *and* the maintained assumption that I have constant risk aversion over this interval of prizes.

This is hardly surprising. My intuitive judgment is fallible. Moreover, I am only approximately constantly risk averse over the interval $-\$50,000$ to $\$150,000$. But, as a normative aid to decision making, I find constant risk aversion a good working hypothesis, and my three CE judgments suggest that basing choices (for gambles in this range) with the utility function $u(x) = -e^{-\lambda \cdot x}$ for λ around 0.00001 will give me choices that are coherent (that is, that obey the five basic properties), have constant risk aversion, and are roughly in accord with the intuitive CE judgments I make.

Specifically, the value $\lambda = 0.00001$ gives me CEs for the three gambles of, respectively, \$49,173, \$4,875, and \$21,907. Table A4.1 provides the three CEs, for four values of λ : $\lambda = 0.00000548, 0.0000402, 0.000013$, and 0.00001 . Note that, as λ gets smaller, my CEs get larger; λ is called the *coefficient of risk aversion*, and the bigger it is, the more risk averse an individual is. Note also how my CEs respond to this parameter; there is little response (over this range of values) for the second gamble and the most dramatic response for the first.

lambda	CE1	CE2	CE3
0.00000548	\$60,003	\$4,932	\$23,293
0.0000402	\$17,183	\$4,501	\$14,115
0.000013	\$43,087	\$4,838	\$21,007
0.00001	\$49,173	\$4,875	\$21,907

Table A4.1. CEs for three gambles, for four levels of (constant) risk aversion.

Table A4.1 gives me the data I need to answer the question, How do I feel about $\lambda = 0.00001$? Essentially, this question is, How comfortable am I with CEs of \$49,000 or so for the first 50–50 gamble, \$4900 or so for the second, and \$22,000 or so for third? Compared to my original estimates of \$60,000, \$4500, and \$21,000, I am obviously fairly happy with the third. But am I certain that I would rather take the first gamble instead of \$55,000? Am I certain that I would take \$4800 over the second? It turns out (not surprisingly) that

I am certain of neither of these, which is why I think $\lambda = 0.00001$ is a good “compromise” value.

Let me say again, *I am not a perfect chooser of gambles under uncertainty*. I expect my intuitive judgment to be somewhat flawed. The power of this approach is that I can look normatively at the five principles, and say, “In this context, I buy them.” And I can look at the property of constant risk aversion for gambles in this range and say, “For me and for this range of prizes, roughly constant risk aversion makes sense.” Then it follows logically that I want my choices to conform to maximization of expected utility, where $u(x)$ takes the form $-e^{-\lambda x}$ for some constant λ (and for x in this range). I can then look at some intuitive judgments, get a rough handle on what λ might be, see how consistent my judgments are, and finally massage my intuitively derived CEs, to come up with more consistent numbers.

If I find the five qualitative properties plus constant risk aversion appealing on qualitative grounds, *and I do*, I am apt to give myself more credit for such qualitative judgment than I am for intuitive quantitative CE judgment, even for simple gambles. So when something has to give, I shade my intuitive quantitative judgment and end up more confident that I make a better choice, in consequence. That is how to use this model normatively. For more on this, see Problem A4.2.

A4.3. What Sort of Behavior Is Sensible?

We discussed the justification for the expected utility model as a normative decision aid in the context of gambles with objectively known probabilities. What about problems with subjective uncertainty?

Recall from Chapter 18 in the text that, as descriptive theory in such circumstances, economists use the expected-utility model, assuming that the decision maker assesses subjective probabilities where necessary and then treats them on par with any objective probabilities that might be present. In other words, aversion to subjective uncertainty is not admitted into most economic models, which, since aversion to subjective uncertainty is a fact of life, is a deficiency of expected utility as a descriptive model.

As a matter of normative theory, we come to a different conclusion. It takes a different set of properties and a different, more general formulation of what is a gamble. But for properties just as reasonable as the five given previously, if you subscribe to the normative desirability of the expanded set of properties, the conclusion is that you should assess probabilities and use them as if they were objective. Uncertainty aversion (avoiding gambles that involve unknown odds simply because the odds are unknown) isn’t normatively sensible.

I do not go into the details here. If you consult a more advanced book on microeconomics (and, as always, I recommend Kreps, *op.cit.*) or choice under uncertainty, look for the theories of choice under uncertainty due to Savage and Anscombe–Aumann.

Framing effects

It should be obvious from the discussion on defeating the zero illusion that, as far as normative theory goes, economics (or, rather, economists) assume that you *do not* want to be fooled by changes in how specific questions are framed. It is one thing—a true thing—to say that people can be fooled into changing decisions they might make, depending on how their options are framed. It is quite another thing—and one that I expect you would not agree to—to say that being fooled in this fashion is desirable. And if you agree that being fooled by framing is undesirable, this merely reinforces the admonitions made last section about being careful how you frame the questions that go into building your utility function.

The certainty and small-probability effects

Similarly, we observed in the text that individual choices can exhibit the certainty and small-probability effects (see Figures 18.5 and 18.6). Since these effects are inconsistent with expected-utility maximization, as asserted in the text, and since I'm advocating expected-utility maximization as normatively desirable, I evidently assert that these are normatively undesirable behavior patterns you would do better to avoid.

In fact, it is not hard to see that, of the five properties, it is Properties 1 and 5 combined that these effects violate. Suppose you prefer Gamble B to Gamble A from Figure 18.5. Property 5 then says you prefer the compound gamble in which you get B with probability 0.9 and \$0 with probability 0.1 to the compound gamble where you get A with probability 0.9 and \$0 with probability 0.1. But the first of these compound gambles reduces by the laws of probability to Gamble D, while the second reduces to Gamble C. Hence, if you subscribe to Properties 1 and 5 and you prefer B to A, you must want to prefer D to C. (You can construct a similar argument for the gambles in Figure 18.6.)

Is Risk Aversion Sensible?

From the list of behavioral phenomena introduced in the text as phenomena observed when people make choices with uncertain consequences, we now assert that framing effects, aversion to subjective uncertainty, the certainty effect, and the small-probability effect are all normatively undesirable. This leaves risk aversion, asymptotic risk neutrality as the scale of the gamble approaches zero, and decreasing risk aversion as wealth increases. Are any of these undesirable? In particular, is risk aversion undesirable?

Students sometimes argue that being risk averse is indeed normatively undesirable, or foolish, in the sense that, if you always pick the gamble with the largest expected value, the law of averages says that eventually

you come out ahead. But this argument is nonsense. First, even if the law of averages applied—and I argue in a moment that it does not always—if gambles today provide a stake for your gambles tomorrow, then you would want to maximize the expected rate of growth of your fortune and not the additive increment obtained from any gamble. These are not the same. But, more important, the law of averages holds that *eventually* someone betting on the highest EMV gambles comes out ahead. How long is eventually? Ten years? Twenty? Ten thousand? And what happens if the person goes bankrupt before the law of averages kicks in? Certain gambles in your life are not repeated five times, let alone the number of times necessary to put your faith in the law of averages; and there is nothing wrong with being risk averse in the face of those risks.

A4.4. Reasons to Be Suspicious

There are, however, good reasons to be suspicious of the normative desirability of following the expected-utility model. Here are two:

Portfolio Effects

Suppose you are engaged in some personal investments. In particular, suppose you hold a portfolio of stocks that includes a number of shares of Ford Motor Company. Your stock broker calls with a suggestion that you speculate on Ford Motor Company call options. If you buy these options, you would make money when your shares of Ford Motor Company are doing well and you would lose money when Ford's share price goes down.

If you followed the procedures outlined in this chapter, you might consider the consequences of this proposed speculation as a gamble and compare the expected utility of this gamble with the expected utility of doing nothing or investing your money in some other fashion.

This is not the right thing to do, however, because the payoff from this particular gamble is positively correlated with the value of your stock portfolio. In times when you are relatively richer, this gamble gives you a good return. In times when you are relatively poorer, this speculation provides poor returns. That is, this risk *compounds* the risk you already face. In such instances, you cannot evaluate this risk alone; you must consider it in conjunction with your entire portfolio of assets.

Investment advisors understand this problem very well. If you did not know better, you would be advised to analyze your entire portfolio of investments as a portfolio and not look at individual investments. But this problem is not limited to investments in financial markets. In most instances where probabilities are subjective, because they are probabilities of

real-world things, the value of the prizes to you depend on the “state” in which the prize is received. For instance, if you are contemplating an entrepreneurial venture, involving the marketing of a new product, success is apt to be correlated to some extent with economic good times. But payoffs from this venture are probably worth more to you in economic bad times; putting it as negatively as possible, if your venture goes bust, you will need a job, and finding a job when the economy is growing slowly or even in a recession is more difficult than when the economy is booming. If you compare two entrepreneurial ventures that have exactly the same probability distributions of payoffs to you, but one pays off more when times are good and the other pays off more when times are bad, the latter is probably more valuable to you.

An extreme example of this concerns insurance. If you purchase, say, fire insurance on your house, the premium you pay exceeds the expected amount of money you will be paid. If you look at this purchase of insurance as an isolated gamble, it will look quite bad; it has a negative EMV and is risky as well. But that is not the way to look at insurance. The point of insurance is that it compensates you in circumstances when compensation is relatively valuable to you, for instance when your house burns down.

What happened to the five properties in these three instances? Why don't they work? None of the five properties is at fault; the entire approach to the problem is flawed. The starting point of this approach to choice under uncertainty is to say that, when you look at a gamble, all that matters to you are the prizes you receive and the probabilities with which they are received. That is incorrect in these instances. In these stories, the “state of nature” affects the value to you of the prizes you might receive. Two gambles with the same prizes and probabilities may look quite different to you, if they give their good payoffs in very different circumstances.

Two approaches can be taken to deal with portfolio problems. You can work out the value of the different prizes in the different possible circumstances the prizes might be received. This works great in theory, but it vastly complicates the assessment of your utility function. Or you can frame the decision problem on a scale large enough that the basic model works. For instance, look at the gamble on Ford call options not in isolation but in terms of your entire investment portfolio. Consider payoffs from insurance policies not in isolation but in terms of your net wealth with and without insurance, in case of fire or some other disaster.

Temporal Resolution of Uncertainty

The second caution has the fancy name of *temporal resolution of uncertainty*. Suppose I offered you three gambles. In the first, you get \$500,000 with

probability $1/2$ and \$0 with probability $1/2$. In the second, you get \$500,000 with probability $1/2$ and \$0 with probability $1/2$. In the third, you get \$500,000 with probability $1/2$ and \$0 with probability $1/2$. No, my word processor didn't get stuck. These gambles are the same in terms of prizes and probabilities, but they differ in another fashion. In each case, if you win the \$500,000, you will get the money precisely nine months from today. In the first gamble, I flip a coin nine months from today and tell you the outcome then. In the second, I flip the coin today in the presence of a reliable witness and tell you the outcome in nine months. In the third, I flip the coin today and tell you the outcome today, although you get the money in nine months.

Most people prefer the third of these "gambles." Having the uncertainty resolve sooner is valuable, because it allows you to plan other activities. For example, if you know that you have \$500,000 coming to you nine months from now, you may change your vacation plans or your employment strategy. If I do not tell you whether you won the money or not for nine months, you have to temporize in decisions you make in the interim.

The point is that these three "gambles" have the same prizes with the same probabilities. The expected utility procedure would necessarily rank them the same, since the expected utility procedure considers only prizes and their probabilities. But, in general, gambles that resolve earlier are more valuable because of the information they bring. This is something the expected utility procedure simply misses; when this effect is important, the expected utility procedure can be badly misleading.¹ In such cases, you must take a longer-term, broader view of the choices you have to make, including in a single model the full set of related decisions you must make.

A third reason to be suspicious, or If this is such a great idea, why haven't you seen it before?

Portfolio effects and problems with temporal resolution of uncertainty can complicate application of the expected utility model, but still, if you are aware of these problems and do not push the expected-utility model where it is unsuitable, the model can be a remarkably powerful aid for making better decisions when you face uncertain consequences.

At this point, a question is inevitable: If this is such a great idea, why haven't you seen it before? Or, put a bit differently, Do real managers ever use this procedure for deciding among the gambles they face?

There are cases in which this precise procedure is practiced. People who

¹ When uncertainty resolves at a later date, this raises other problems for the expected utility procedure. Essentially, late-resolving uncertainty makes property 5 dubious. This gets very technical, so I do not go into details here. If you are interested, see Chapter 12 of D. Kreps, *Notes on the Theory of Choice* (Boulder, CO: Westview Press, 1988).

ply decision analysis make healthy salaries consulting on the uses of this technique. But, if you do not remember seeing this technique previously, you probably have not slept through meetings or failed to read memos you were supposed to read. This is not a widely used decision aid.

The reason for this—and a third and final reason to be suspicious of the expected utility model, applied in specific situations—is that this technique assumes that the risks involved in the gamble are borne by a single individual. Insofar as many risks taken by managers are risks taken on behalf of some firm or agency, the personal risk attitudes of those on whose behalf the decision is made are important, not those of the manager. Risks taken by firms, government agencies, and other organizations are often spread among many people. Risk sharing of this sort dilutes the risk in ways that we explore in Chapter 19 of the text, so that risk aversion no longer plays an important role. Or, more precisely, risk aversion plays a role, but it is risk aversion applied to something other than the immediate riskiness of the gambles being considered. *You do not see this procedure practiced much in the real world, because real-world settings are rich in opportunities for risk sharing.*

Bottom line: As a normative decision maker, you must understand those opportunities, because it is often best (for you) to take advantage of them. This model, used descriptively, in the fashion of Chapter 18 in the text, gives us starting point for understanding. But it is only the starting point; Chapter 19 takes the next step.

Executive Summary

(I provide an executive summary, since some instructors may wish to include this material as a “regular chapter” in their courses. I would do so at the Stanford GSB if we had a few more sessions.)

- The expected utility model, used descriptively in Chapter 18 of the text, is repackaged in this chapter as a normative decision aid, intended to help you make better decisions when facing a choice problem with uncertain consequences.
- The key to adopting this decision aid is to contemplate whether you agree that the five properties given on pages C303-4 are desirable for your preference judgment. If you find them desirable normatively, it follows logically that you want your decisions to conform to the expected utility model, for some utility function.
- The problem then is to assess your personal utility function. Using the simplest possible judgment of this sort (comparing gambles with two equally likely prizes to sure things) you can rough in your utility function. And a variety of techniques exist for refining the utility function so assessed.
- In some cases, the five properties are not normatively attractive, at least for

simple-minded application. These include cases in which there are substantial problems with correlated risks (portfolio problems) and cases with temporal resolution of uncertainty.

- Finally, if you face risk, you should consider the options you have for spreading and sharing that risk. So please read Chapter 19!

Problems

Here are two problems for you to try. I don't supply an answer to Problem A4.1, because the answer you give is completely subjective. The solution to Problem A4.2 follows immediately after the statement of the problem.

A4.1 Go back to the question with which the chapter began. Suppose you were offered a choice from the five gambles in Figure A4.1.

(a) Try to rank these five according to your unaided intuition about how good each is for you. If you can, figure out which in your ordering are "close" and where the significant gaps in quality (as far as you are concerned) occur.

(b) Assess (roughly) your own utility function for gambles with prizes between $-\$7500$ and $\$15,000$, using the techniques discussed in this chapter.

(c) Use this utility function to evaluate the five gambles and find their certainty equivalents as far as you are concerned. Compare these with the answers you gave in part a. Which technique, unaided choice or the technique of parts b and c, do you find more satisfactory?

A4.2 Students sometimes object to the technique advanced for finding one's coefficient of risk aversion, once the judgment is made that, over some range, the individual has roughly constant risk aversion. More specifically, students sometimes object to the rather slapdash emergence of $\lambda = 0.00001$ for me, as described in the chapter.

But does the precise value of λ make a difference? Suppose I must choose from among the five gambles in Figure A4.1. Suppose I decide that my level of risk aversion is constant, more or less, for the range $-\$50,000$ to $\$150,000$, which more than covers the range of prizes of the five gambles. Then, by the methods discussed in the chapter, I decide that my coefficient of risk aversion was somewhere between 0.000005 and 0.000015 , with 0.00001 as a "good compromise." How much would it matter if I had settled on 0.000012 or 0.000008 ? To answer this question, take the five gambles in Figure A3.1 and see how a decision maker would evaluate them, if the decision maker has constant risk aversion (for the range of prizes in these gambles) with a coefficient of risk aversion between 0.000005 and 0.000015 .

Solution to Problem A4.2

I created the following spreadsheet to answer Problem A4.2. (You should try to replicate it.)

	A	B	C	D	E	F	G	H	I	J
1			coefficient of risk aversion			0.000005	0.000008	0.00001	0.000012	0.000015
2										
3			prizes	probs				UTILITY LEVELS		
4										
5		gamble A	\$6,000	1		-0.9704455	-0.9531338	-0.9417645	-0.9305309	-0.9139312
6										
7		gamble B	\$10,000	0.8		-0.9512294	-0.9231163	-0.9048374	-0.8869204	-0.860708
8			-\$5,000	0.2		-1.0253151	-1.0408108	-1.0512711	-1.0618365	-1.0778842
9										
10		gamble C	\$10,000	0.5		-0.9512294	-0.9231163	-0.9048374	-0.8869204	-0.860708
11			\$3,800	0.5		-0.9811794	-0.9700574	-0.9627129	-0.9554241	-0.9445941
12										
13		gamble D	\$13,000	0.3		-0.9370675	-0.9012253	-0.8780954	-0.8555592	-0.8228347
14			\$4,000	0.5		-0.9801987	-0.9685066	-0.9607894	-0.9531338	-0.9417645
15			\$2,000	0.2		-0.9900498	-0.9841273	-0.9801987	-0.9762857	-0.9704455
16										
17		gamble E	-\$7,500	0.07		-1.038212	-1.0618365	-1.0778842	-1.0941743	-1.1190723
18			\$1,000	0.23		-0.9950125	-0.9920319	-0.9900498	-0.9880717	-0.9851119
19			\$6,000	0.38		-0.9704455	-0.9531338	-0.9417645	-0.9305309	-0.9139312
20			\$15,000	0.32		-0.9277435	-0.8869204	-0.860708	-0.8352702	-0.7985162
21										
22		EXPECTED UTILITIES								
23				gamble A		-0.9704455	-0.9531338	-0.9417645	-0.9305309	-0.9139312
24				gamble B		-0.9660466	-0.9466552	-0.9341242	-0.9219037	-0.9041432
25				gamble C		-0.9662044	-0.9465869	-0.9337752	-0.9211722	-0.902651
26				gamble D		-0.9692295	-0.9514463	-0.9398631	-0.9284918	-0.9118218
27				gamble E		-0.9671749	-0.9485013	-0.9364604	-0.9247369	-0.9077298
28										
29		CERTAINTY EQUIVALENTS								
30				gamble A		\$6,000	\$6,000	\$6,000	\$6,000	\$6,000
31				gamble B		\$6,909	\$6,853	\$6,815	\$6,776	\$6,718
32				gamble C		\$6,876	\$6,862	\$6,852	\$6,842	\$6,828
33				gamble D		\$6,251	\$6,221	\$6,202	\$6,183	\$6,154
34				gamble E		\$6,675	\$6,609	\$6,565	\$6,521	\$6,454
35										

For $\lambda = 0.000005, 0.000008, 0.00001, 0.000012,$ and $0.000015,$ I compute EUs and CEs for each of the five gambles. Gamble B wins for $\lambda = 0.000005,$ but Gamble C is only \$33 behind; Gamble C wins for all the other values of $\lambda.$ I'm going with Gamble C.