Chapter 12 Material

The solutions to problems from Chapter 12 are presented. In addition, an exercise is presented and solved concerning optimal pricing when costs are driven by an experience curve.

12.1 (a) This is the sort of exercise done multiple times in Chapter 11, so I'll give only the highlights: To be cost minimizing at the prices given, the ratios of k to l to m must be

$$96k = 48l = 6m \quad \text{or} \quad 16k = 8l = m.$$

(To be overly pedantic: These are the right ratios for the given prices. As the relative factor prices change, the optimal ratios change.) So to find $\mathrm{TC}(x)$, since we need

$$x = k^{1/2} l^{1/3} m^{1/6},$$

we can substitute $16k$ for m and $2k$ for l and get

$$x = k^{1/2}(2k)^{1/3}(16k)^{1/6} = 2^{1/3}16^{1/6}k = 64^{1/6}k = 2k.$$

Hence,

$$k = \frac{x}{2}, l = x, \text{and } m = 8x, \quad \text{so that} \quad \mathrm{TC}(x) = 48\frac{x}{2} + 16x + 8x = 48x.$$

(b) With inverse demand $P(x) = 192 - x$, marginal revenue is $192 - 2x$, so profit maximization occurs at

$$192 - 2x = 48 \quad \text{or} \quad x = 72.$$

At this level of output, the status-quo levels of inputs are $k = 36, l = 72,$ and $m = 576$.

(c) In the short run, from this status-quo point, with k fixed at 36, the short-run production function is $x = 36^{1/2}l^{1/3}m^{1/6} = 6l^{1/3}m^{1/6}$. The short-run cost minimizing plan (at the given prices) has $48l = 6m$, or $8l = m$, so substituting $8l$ for m gives $x = 6l^{1/3}(8l)^{1/6} = 6\sqrt{2}l^{1/2}$, and so $x^2 = 72l$, or

$$l = \frac{x^2}{72} \text{ and } m = \frac{x^2}{9} \quad \text{so that} \quad \text{SRTC}(x) = 1728 + 16\frac{x^2}{72} + \frac{x^2}{9} = 1728 + \frac{x^2}{3}.$$

(d) When inverse demand shifts to $P(x) = 200 - x$, marginal revenue shifts to $\text{MR}(x) = 200 - 2x$. In the short-run, the profit-maximizing level of x is where

$$200 - 2x = \frac{2x}{3} \quad \text{or} \quad 200 = \frac{8x}{3} \quad \text{or} \quad x = 75,$$

at which level of prouction (with k fixed at 36) the firm uses $l = 78.125$ and $m = 625$.

And in the long run, when k can vary, the profit-maximizing level of x is where

$$200 - 2x = 48 \quad \text{or} \quad x = 76,$$

at which point $k = 38, l = 76$, and $m = 608$.

12.2 (a) See figure S12.1. There isn't much of a "gap" between the LR and the SRTC functions, but the shapes are correct.

(b) I preface this answer by reiterating: This is not an easy question to answer. If you are (even) able to follow the answer I'm about to give, you are doing very well in your understanding of these matters.

Think of how the rule for cost minimization is rationalized: You have $r_i/\text{MPP}_i \leq r_j/\text{MPP}_j$ if $y_i > 0$, because, otherwise, you could substitute a small amount of input j for input i in the ratio $1/\text{MPP}_j$ to $1/\text{MPP}_i$, which would keep the production level the same and decrease costs. In the situation described, the input l is "semi-fixed" at the status-quo level of 64: You can raise it, but if you raise it (past 64) then it costs (on the margin) \$6 per unit. And you can lower it, but doing so saves (on the margin) only \$2 per unit.

So, stating from the status-quo position of $l = 64$ and $m = 512$, so that $x = 16$, suppose you want to raise x. You can do this by increasing m or

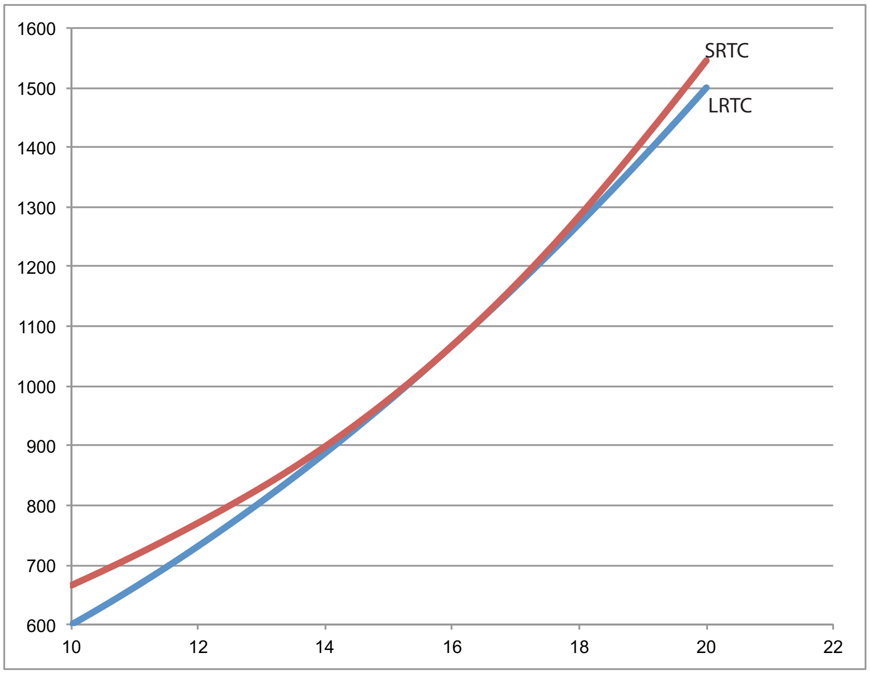

Figure S12.1. The LR and SRTC functions for Story 2 from page 286 in the text.

l or both. To increase x by a small amount, say by ϵ, takes an increase in m of ϵ/MPP_m and so costs $\$1\epsilon/\mathrm{MPP}_m$. To increase x by ϵ by increasing l takes an increase in l of ϵ/MPP_l and so raises costs by $\$6\epsilon/\mathrm{MPP}_l$. But at $l = 64$ and $m = 512$, the starting position, we know that $4/\mathrm{MPP}_l = 1/\mathrm{MPP}_m$. (Why do we know this? Because this is the equation that got us the 8 to 1 ratio at input prices \$4 and \$1.) So, on the margin, it is cheaper to increase output by increasing only m, leaving l at 64. Eventually, however, as we keep increasing m, we are driving MPP_m down. And, at some point, as we continue to increase m, we'll reach a point where $\$6/\mathrm{MPP}_l = \$1/\mathrm{MPP}_m$.

When does this happen? For l fixed at 64, at what level of m does $\$6/\mathrm{MPP}_l = \$1/\mathrm{MPP}_m$? If you do the math, you'll discover that $\$6/\mathrm{MPP}_l = \$1/\mathrm{MPP}_m$ when the ratio of m to l is 12 to 1. Since l is sitting at 64, this happens when m gets up to $12 \times 64 = 728$, at which point x has increased to $64^{1/6}728^{1/3} = 18.3145$.

And then, for further increases in x, we keep the ratio of m to l at 12 to 1; now adding l, even at the higher SR cost, is cost minimizing.

So what does the SRTC function look like for increases in x from the status-quo level of 16? From 16 up to 18.3145, we are getting more x by adding m only, and we know (from Story 1 on page 285) that this gives a SRTC function of $556 + x^3/8$. But beyond $x = 18.3145$, the cost minimizing plan is to use

m to l in the ratio 12 to 1. That is, $m = 12l$. Since $x = m^{1/3}l^{1/6}$, this gives $x = (12l)^{1/3}l^{1/6} = 12^{1/3}l^{1/2}$, or

$$l = \frac{x^2}{12^{2/3}} = 0.19079x^2 \quad \text{hence} \quad m = 12 \times 0.19079x^2 = 2.289x^2.$$

And this gives, for $x > 18.3145$, a total cost of

$$300 \quad + \quad \$4 \times 64 \quad + \quad \$6(0.19079x^2 - 64) \quad + \quad \$1 \times 2.289x^2.$$

To explain, this is the LR fixed cost of \$300 plus the cost of the first 64 unilts of l, plus the cost of the units of l in excess of 64, plus the cost of the m. Simplify this expression, and you get $172 + 3.434143x^2$.

This gives the total cost function supplied in the text for $x \geq 16$. For $x \leq 16$, you apply similar logic: For "small" decreases from 16, you only decrease m, but eventually, it becomes economical to decrease both m and l. If you've followed this answer this far, you are smart enough to do this on your own, so have fun.

12.3 For the firm to produce $x \geq 16$, it must use $4x$ units of l and $32x$ units of m. In the short run, the 32 units of m cost \$$32x$. And the $4x$ units of l cost \$4 $\times$ 64 for the first 64 units and \$6 $\times$ $(4x - 64)$ for the "extra" units of l. Hence the short-run total cost for $x \geq 16$ is

$$32x + 4 \times 64 + 6 \times (4x - 64) = 56x - 128.$$

And to produce $x \leq 16$, again the firm requires $4x$ units of l and $32x$ units of m. The $32x$ units of m cost \$$32x$. As for the l, the firm pays \$4 for each of the $4x$ units of l that it employs, and it pays \$2 for each of the $64 - 4x$ units that it lays off or furloughs. Hence its short-run total cost is

$$32x + 16x + 2 \times (64 - 4x) = 40x + 128.$$

(You might want to graph the LR and SR total-cost functions and, on a separate graph, the LR and SR marginal-cost functions for this problem.)

12.4 If the price of l rises to \$6 per unit, the cost-minimizing mix of the two inputs shifts. It had been eight m to one l; now the governing equation is

$$\$1/\text{MPP}_m = \$6/\text{MPP}_l \quad \text{which gives} \quad 3m = 36l \quad \text{or} \quad m = 12l.$$

Hence LTRC is found by substituting $12l$ for m in $x = l^{1/6}m^{1/3}$ to get

$$x = l^{1/6}(12l)^{1/3} = 2.2894l^{1/2} \text{ or } l = x^2/2.2894^2 = 0.19079x^2,$$

hence $m = 2.28943x^2$. And so, the long-run total-cost function is

$$300 + \$1 \times 2.28943x^2 + \$6 \times 0.19079x^2 = 300 + 3.434143x^2.$$

(If you worked you way through Story 2 on page 286 or the solution to Problem 12.2, the constant 3.43143 should be familiar. But the fixed cost is different. Can you see why?)

As for the short run, the firm is stuck with $l = 64$, so to produce x it must set m so that

$$x = x^{1/3}64^{1/6} = 2m^{1/3} \quad \text{or} \quad m = x^3/8.$$

So the short-run total-cost function is

$$\$300 + \$6 \times 64 + \$1 \times x^3/8 = 684 + x^3/8.$$

Before finishing the problem, let me plot the new SR and LR total-cost functions. See Figure S12.2.

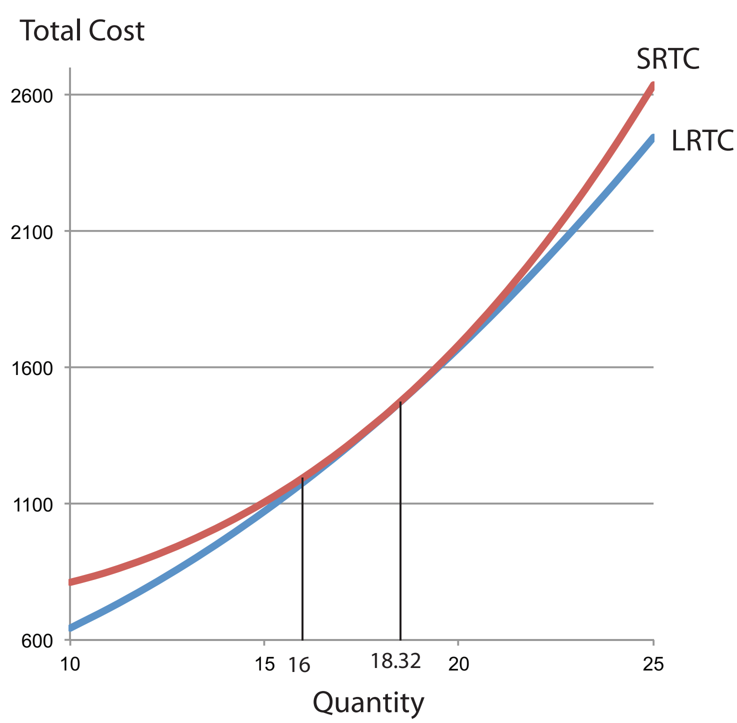

Figure S12.2. LR and SRTC for Problem 12.4, after a change in the price of input l.

This looks just like the other pictures we've drawn, except for one thing: The place where long-run and short-run total costs are equal is *not* the status-quo point but at a higher level, the level $x = 18.32$ (approximately). Why is this?

This happens because, at the new price for l of \$6 per unit, the sold status-quo ratio of $8m$ to l is no longer long-run cost minimizing. To be cost minimizing (in the long run), the ratio has to be 12 to 1. Since l is fixed at 64 in the short run, it takes $m = 768$ to achieve the new "long-run efficient" ratio. And $m = 786$ (with $l = 64$) produces (approximately) $x = 18.32$.

To finish up: With the new price for l, and the SRTC function $684 + x^3/8$, the SRMC function is $3x^2/8$, which equals marginal revenue where

$$3x^2/8 = 160 - 4x, \quad \text{which, solving for } x\text{, gives} \quad x = 16.$$

In the short run, the firm makes no change whatsoever. In fact, when you think it through, this is not surprising. SRMC, $3x^2/8$, does not depend at all on the price of l, since l is a fixed factor of production in the short run. SRTC is affected by the increased price of l; the SR fixed cost that used to be \$556 is now \$684, the difference being the extra \$2 per unit of l times the 64 units of l that must be purchased. And is SR*M*C is the same as before, the profit-maximizing quantity is the same.

But LRMC has changed; in the long run, the firm does react. LRMC = MR is

$$6.868286x = 160 - 4x \quad \text{or} \quad x = 14.7217,$$

at which $l = 0.19079x^2 = 41.35$ and $m = 12l = 496.198$. It is not surprising that x falls, or that the more expensive input l falls. But m also decreases a bit.

If you've digested all this, a good drill is to try a similar problem where the price of the un-fixed input shifts. For instance, suppose that the price of l stays at \$4, but the price of m rises to \$2. How does the firm react in this case, both in the short run and in the long run? (This is posed in Review Problems III as Problem III.8, so if you give it a shot, you can check your work there.)

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We finish the *Online Supplement* material for Chapter 12 with some further exploration of the experience curve. The first step is to derive the total-cost function given in the text.

Deriving the Formula for the Experience Curve

In the text, I asserted that a pure experience-curve total cost function takes the form

$$\mathrm{TC}(x, X) = \frac{c_1[(X+x)^\beta - X^\beta]}{\beta}, \quad \text{for } \beta = \frac{\ln(\gamma)}{\ln(2)} + 1,$$

where x is output in the given period and X is cumulative prior output. The constant γ is the "slope" of the experience curve, in the sense that costs fall to γ of their previous level every time cumulative production doubles; that is, the sequence c_n of unit cost figures has the property $c_{2n} = \gamma c_n$ for all n.

To derive this formula, we note first that, with an experience-curve effect, the sequence of unit costs takes (roughly) the form $c_n = c_1 n^{\beta-1}$, for some constant β. (In fact, this is really the defining characteristic of an experience-curve effect. Simply knowing that costs fall by γ for every doubling of cumulative output does not quite pin things down.) Now, the property $c_{2n} = \gamma c_n$ tells us that $2^{\beta-1} = \gamma$, which you can solve for β in terms of γ, to get the formula $\beta = [\ln(\gamma)/\ln(2)]+1$. To find $c_{X+1} + \ldots + c_{x+X}$, we approximate the sum with $\int_x^{X+x} c_1 y^{\beta-1} \mathrm{d}y$, which when evaluated gives the formula we provided.

Pricing Down the Experience Curve?

The following is a fairly long, involved "problem," so long and involved that I left even its statement for the *Online Supplement*. Despite its length and complexity, I recommend it highly as a great way to wrap up the discussion of the experience curve and the first ten chapters of the book, as it contains a very nice final example of thinking intelligently about marginal this and marginal that.

As noted in the text, in the late 1970s and early 1980s, the big "strategy" fad in manufacturing was the experience curve. High-priced consulting firms pitched their clients the notion that a winning strategy was to become the market-share leader in the early days of a product, to become the "experience leader" and have the lowest unit costs. Eventually the market-share leader would have such a cost advantage over rivals that all the rivals would be driven out—or at least become docile—and the leader would reap large profits.

How do you become the market share leader? By selling your product at very low cost. *Buy market share!* was the short-form version of this strategy.

Firms were exhorted to price down the experience curve, keeping margins no more than razor thin, lest someone take away some of your market share and, eventually, beat your brains out with their experience-based cost advantages. These ideas mutated from business strategy to trade policy, when NICs (newly industrialized countries) put in place tariff barriers to "protect" their infant industries from the ravages of more experienced global competitors.

We are not ready to address the merits of this bit of business strategy, because until we get to Chapter 22, we are in no position to discuss seriously competition among industrial rivals. Even after Chapter 22, questions of appropriate business strategy when firms have experience-curve technologies are not easy to answer. But we can, at least, take on a simpler question: Set aside all notions of rivals and competition and buying market share. If a single producer faces a stable market with an experience-curve-driven cost structure, should the producer price down the learning curve?

Imagine a firm that develops a new product, with a clear, well-defined market: If the firm produces x units of the good for sale in a given quarter, the price per unit that the firm receives is $P(x) = 1000 - 0.08x$. The market for this item lasts for precisely 5 years (20 quarters), and the firm must decide on production levels for each of the 20 quarters; let x_1 denote the level of production in quarter 1, x_2 the level of production in quarter 2, and so on.

The complicating feature in this problem is that the firm's production technology exhibits a substantial experience-curve effect. Specifically, the cost to produce the nth cumulative unit is approximately 80% of the cost of producing the $n/2$th cumulative unit. The firm has produced five prototype units, so the first unit it produces in the first quarter of sales is its sixth.

Cost engineers at this firm, who are very good at these sorts of problems, have been able to give Marketing the following formulas: Suppose that in quarter 1 the firm produces x_1 units. Then its total costs of production in that quarter are

$$\$22166\left[(x_1 + 5)^{0.6767} - 5^{0.6767}\right].$$

If it goes on to produce x_2 units in the second quarter, its total costs for the second quarter are

$$\$22166\left[(x_2 + x_1 + 5)^{0.6767} - (x_1 + 5)^{0.6767}\right],$$

and so on. Write X_{n-1} for the *total* or *cumulative* number of units produced through period $n-1$, including the five prototypes; that is,

$$X_{n-1} = 5 + x_1 + x_2 + \ldots + x_{n-1}.$$

Then, if the firm goes on to produce an additional x_n units in quarter n, its total costs for quarter n are

$$\$22166\left[(x_n + X_{n-1})^{0.6767} - (X_{n-1})^{0.6767}\right].$$

What values of x_1, x_2, ..., and x_{20} maximize the sum of the firm's cash flows over the 20 quarters? (Do not discount the cash flows, at least not at the start.) In particular, since costs fall as more units are made, this means costs in the last quarter are below costs in early quarters. Since demand and therefore marginal revenue do not shift, does this imply that the firm should produce less in early years, so that the prices it charges fall as costs come down the experience curve?

To investigate this question, build a spreadsheet. In column B, simply record the quarter number. (Leave a few blank rows on top of the spreadsheet.) In column C, for each quarter put in the quantity to be produced in that year. In column D, use the inverse demand function to find the price the firm gets in each quarter as a function of the number in column C. In column E, put total revenue for the quarter (column C times column D). In column F, put the cumulative output to the end of that quarter. Remember the five prototypes. In column G, compute the total cost incurred during the quarter, using the formula given previously. In column H, compute the profit for the quarter (column E minus column G). Then, in a cell below this table, add the 20 profit figures.

So there is no misunderstanding, Figure S12.3 shows the desired spreadsheet, for the values $x_1 = 250$, $x_2 = 500$, $x_3 = 750$, and so on. (Note that the particular production plan I filled in gives total profit of \$3,554,388.) First try to replicate this spreadsheet. (If you wish simply to copy it, it is named EXPERIENCE-CURVE.)

Now for the fun: try different values for x_1 through x_{20} to see how high you can push the 5-year profit figure. To give you a goal, I got them as high as \$8,443,585. Use Solver if you wish, although I warn you that Solver does not always work, at least not from all starting values, and this is a case where you might learn more by playing around with the numbers a bit.

In the optimal production plan, $x_1 = x_2 = \ldots = x_{20}$. Can you figure out why this is? What about MC = MR in this problem?

	A	B	C	D	E	F	G	H
4								
5		quarter #	quantity	price	revenue	cum prod	cost	profit
6						5		
7		1	250	$ 980.00	$ 245,000	255	$ 876,440	$ (631,440)
8		2	500	$ 960.00	$ 480,000	755	$ 1,021,929	$ (541,929)
9		3	750	$ 940.00	$ 705,000	1505	$ 1,168,516	$ (463,516)
10		4	1000	$ 920.00	$ 920,000	2505	$ 1,289,672	$ (369,672)
11		5	1250	$ 900.00	$ 1,125,000	3755	$ 1,393,594	$ (268,594)
12		6	1500	$ 880.00	$ 1,320,000	5255	$ 1,485,258	$ (165,258)
13		7	1750	$ 860.00	$ 1,505,000	7005	$ 1,567,729	$ (62,729)
14		8	2000	$ 840.00	$ 1,680,000	9005	$ 1,643,019	$ 36,981
15		9	2250	$ 820.00	$ 1,845,000	11255	$ 1,712,522	$ 132,478
16		10	2500	$ 800.00	$ 2,000,000	13755	$ 1,777,243	$ 222,757
17		11	2750	$ 780.00	$ 2,145,000	16505	$ 1,837,938	$ 307,062
18		12	3000	$ 760.00	$ 2,280,000	19505	$ 1,895,185	$ 384,815
19		13	3250	$ 740.00	$ 2,405,000	22755	$ 1,949,442	$ 455,558
20		14	3500	$ 720.00	$ 2,520,000	26255	$ 2,001,077	$ 518,923
21		15	3750	$ 700.00	$ 2,625,000	30005	$ 2,050,388	$ 574,612
22		16	4000	$ 680.00	$ 2,720,000	34005	$ 2,097,624	$ 622,376
23		17	4250	$ 660.00	$ 2,805,000	38255	$ 2,142,994	$ 662,006
24		18	4500	$ 640.00	$ 2,880,000	42755	$ 2,186,674	$ 693,326
25		19	4750	$ 620.00	$ 2,945,000	47505	$ 2,228,817	$ 716,183
26		20	5000	$ 600.00	$ 3,000,000	52505	$ 2,269,551	$ 730,449
27								
28						sum of quarterly profits		$ 3,554,388

Figure S12.3. An experience-curve spreadsheet.

Then, to bring a bit more reality into the problem, redo the analysis where you try to maximize the discounted sum of cash flows, discounting at (say) 2.5% per quarter. What does the optimal production plan look like?

As noted already, we are not ready to think formally about how competition in the product market might affect the results of this simple exercise. But you might wish to think about this informally; the analysis I am about to offer has a few things to say on this.

Analysis: Pricing Down the Experience Curve?

Figure S12.4 illustrates the production plan with a total profit of $8,443,585. Note that the level of production is the same in each period, 3765.3 units per quarter. This means that the price per unit, $698.78, does not change. The firm does not price down the experience curve. Instead it sets and sticks to a price, taking substantial losses in the first year ($4.3 million) to make substantial profits in later years.

This is the optimal production plan. If you found an alternative that beats my plan by more than round-off error, you made a mistake in setting up

	A	B	C	D	E	F	G	H
4								
5		quarter #	quantity	price	revenue	cum prod	cost	profit
6						5		
7		1	3765	\$ 698.78	\$ 2,631,098	3770.29	\$ 5,766,167	\$ (3,135,069)
8		2	3765	\$ 698.78	\$ 2,631,098	7535.58	\$ 3,486,169	\$ (855,071)
9		3	3765	\$ 698.78	\$ 2,631,098	11300.9	\$ 2,940,038	\$ (308,940)
10		4	3765	\$ 698.78	\$ 2,631,098	15066.2	\$ 2,633,371	\$ (2,273)
11		5	3765	\$ 698.78	\$ 2,631,098	18831.5	\$ 2,426,527	\$ 204,571
12		6	3765	\$ 698.78	\$ 2,631,098	22596.7	\$ 2,273,473	\$ 357,625
13		7	3765	\$ 698.78	\$ 2,631,098	26362	\$ 2,153,606	\$ 477,491
14		8	3765	\$ 698.78	\$ 2,631,098	30127.3	\$ 2,056,042	\$ 575,056
15		9	3765	\$ 698.78	\$ 2,631,098	33892.6	\$ 1,974,379	\$ 656,719
16		10	3765	\$ 698.78	\$ 2,631,098	37657.9	\$ 1,904,560	\$ 726,538
17		11	3765	\$ 698.78	\$ 2,631,098	41423.2	\$ 1,843,862	\$ 787,236
18		12	3765	\$ 698.78	\$ 2,631,098	45188.5	\$ 1,790,380	\$ 840,718
19		13	3765	\$ 698.78	\$ 2,631,098	48953.8	\$ 1,742,730	\$ 888,368
20		14	3765	\$ 698.78	\$ 2,631,098	52719.1	\$ 1,699,880	\$ 931,218
21		15	3765	\$ 698.78	\$ 2,631,098	56484.4	\$ 1,661,040	\$ 970,058
22		16	3765	\$ 698.78	\$ 2,631,098	60249.7	\$ 1,625,595	\$ 1,005,503
23		17	3765	\$ 698.78	\$ 2,631,098	64014.9	\$ 1,593,056	\$ 1,038,042
24		18	3765	\$ 698.78	\$ 2,631,098	67780.2	\$ 1,563,028	\$ 1,068,070
25		19	3765	\$ 698.78	\$ 2,631,098	71545.5	\$ 1,535,191	\$ 1,095,907
26		20	3765	\$ 698.78	\$ 2,631,098	75310.8	\$ 1,509,277	\$ 1,121,821
27								
28						sum of quarterly profits		\$ 8,443,585
29								

Figure S12.4. The optimal production plan.

your spreadsheet. Here is the logic that tells me this and allows us to derive the answer analytically.

Let x_1, x_2, through x_{20} be the levels of production in each of the 20 quarters. For total profit over the 20 quarters, we have the formula

$$\begin{aligned}\pi(x_1, \ldots, x_{20}) =& x_1 P(x_1) + x_2 P(x_2) + \ldots + x_{20} P(x_{20}) \\ & - 22166[(5 + x_1 + x_2 + \ldots + x_{20})^{0.6767} - 5^{0.6767}],\end{aligned}$$

where $P(x) = 1000 - 0.08x$. Let me explain this: First, we add the 20 revenue terms, one for each quarter. The last term is the total cost over the 20 quarters. We get this by writing down the total cost for quarter 1,

$$22166[(5 + x_1)^{0.6767} - 5^{0.6767}],$$

adding the cost for quarter 2,

$$22166[(5 + x_1 + x_2)^{0.6767} - (5 + x_1)^{0.6767}],$$

and so on. The point is that the term $22166(5 + x_1)^{0.6767}$ which is added for the first quarter is subtracted for quarter 2, and similarly for like terms in quarters 3 through 20, leaving the total cost expression shown.

Let me state that another way. In this "pure experience-curve" technology, each unit produced has a production cost c_z regardless of when it is produced, depending only on its cumulative number z in the sequence of production. The formula given to Marketing by Production may hide this, but it is true. If the firm makes X_{n-1} units in the first $n-1$ quarters, so the first unit made in quarter n is number $5 + X_{n-1} + 1$, and if the firm goes on to make x_n units in this quarter, then its total costs in this quarter are

$$c_{5+X_{n-1}+1} + c_{5+X_{n-1}+2} + \ldots + c_{5+X_{n-1}+x_n}.$$

Since we do not discount revenues or costs, the total cost incurred by the firm in the 20 quarters is just the sum of the costs of the units made, or

$$c_6 + c_7 + \ldots + c_{x_1+x_2+\ldots+x_{20}},$$

which (those smart folks in Production tell us) is

$$2216[(5 + x_1 + \ldots + x_{20})^{.6767} - 5^{.6767}].$$

Knowing that the (undiscounted) sum of the costs of these units is given by this formula is a bit of magic, unless you understood the derivation of the formula given just before the start of this problem. While the formula might be magic, the idea that undiscounted total costs depend only on the total amount produced and not on how that production is divided among the quarters, well, that is a basic property of pure experience-curve technology costs.

To maximize profit, we take partial derivatives in x_1 through x_{20}, and simultaneously set each equal to 0. Rewriting $x_1 P(x_1)$ as $1000x_1 - 0.08(x_1)^2$, the partial derivative in x_1 is

$$\begin{aligned} 1000 - 0.16x_1 - (22166)(0.6767)[5 + x_1 + x_2 + \ldots + x_{20}]^{0.6767-1} &= \\ 1000 - 0.16x_1 - 15000[5 + x_1 + x_2 + \ldots + x_{20}]^{-0.3233}&. \end{aligned}$$

Setting this equal to 0 gives us

$$x_1 = \frac{\left(1000 - 15000[5 + x_1 + x_2 + \ldots + x_{20}]^{-0.3233}\right)}{0.16}.$$

Similarly, we will get

$$x_2 = \frac{\left(1000 - 15000[5 + x_1 + x_2 + \ldots + x_{20}]^{-0.3233}\right)}{0.16}$$

when we set the partial derivative with respect to x_2 equal to 0, and so on.

Note that the right-hand sides of the two previous equalities are the same. Therefore the two left-hand sides must be equal, or $x_1 = x_2$. The same applies for all the other quarterly production levels, or $x_1 = x_2 = \ldots = x_{20}$. And this in turn means that, if x is the common value of these 20 terms, x must solve the single equation

$$x = \frac{1000 - 15000[5 + 20x]^{-0.3233}}{0.16}.$$

To find this value of x, I got out Excel and created two entries. The first was a value for the variable x; the second computed

$$\frac{1000 - 15000[5 + 20x]^{-0.3233}}{0.16} - x.$$

To find a value of x for which the second term is 0, use Solver or hunt for the solution by successive approximation: Whichever you try, you will come to the answer $x = 3765.3$ plus or minus some round-off error.

Actually, things are not that easy. Do this by hand and you will find two different answers, $x = 245\pm$ and $= 3765.3\pm$. If you use Solver, starting from some values takes you to one solution while starting from others takes you to the other. By plugging these values back into the spreadsheet, you find that the second gives the higher profit. But, if you didn't know that there are two solutions and just used Solver, you might come up with just the first.

This is a lot of math, but the intuition is really quite simple. Suppose that, in any year, the firm decides to produce one additional unit. We know what this means in terms of marginal revenue: If the firm is already producing x units that year, its marginal revenue is $1000 - 0.16x$. The marginal cost of an additional unit of production, no matter what year it is produced, is just the marginal cost of the last unit produced in the last period. In this model of production, speed of production is irrelevant. The total costs to the firm of producing 10,000 units the first year and none thereafter are the same as the costs of producing 2,000 in each of the 5 years, and the marginal cost is just the derivative of total costs, or $15000(5 + X)^{-0.3223}$, where X is the total amount produced in the 5 years. So the firm wants to choose x_1 through x_{20} so that

$$1000 - 0.16x_n = 15000(5 + X)^{-0.3223} = 15000(5 + 20x_n)^{-0.3223},$$

or marginal revenue equals marginal cost, for $n = 1, 2, \ldots, 20$. The fancy part here is that marginal cost depends on the total amount produced over 5 years and not on the rate of production in any particular quarter.

That is the theory. What about the facts? In fact, we do not see firms whose production processes have an experience-curve structure pricing in this fashion: producing the same amount each year over the horizon of the product, charging the same price year after year, taking enormous losses early on. Instead, when products have an experience-curve production structure, we often see prices declining as time passes and experience builds up. And we see profit (and profit margin) rising as time passes. There are several reasons for this, some of which can be put into our model and some of which are more complex:

1. In the simple model of costs we created, the cost to produce any unit depends only on the number of units produced prior to this one. The rate of production does not affect costs in the slightest. Compare this with the "standard model," in which the cost of production depends on the rate, where (with rising marginal costs) faster rates mean higher marginal costs. Both are models, and neither is completely accurate. In particular, while many production processes exhibit the basic experience-curve effect (costs declining with cumulative output), it is not true that costs are independent of rate. If Boeing tried to produce 10 times the number of 747s in January as it did in December, its average unit costs would probably rise, even though there is a tendency for its costs to fall with experience. Indeed, part (only part) of the reason that costs fall with experience is that production processes can be speeded up, lowering labor time and hence labor costs per unit. Insofar as this is what drives down unit costs, it can be expensive for a firm to maintain a flat rate of production; it is more efficient (in terms of costs) to increase the speed of production as experience is gained. Another (big) part of the cost savings from experience comes in improved product and process design. Insofar as these improvements must be built into capital equipment (say, with the redesign of jigs and fixtures), it can become (cost) efficient to produce at lower speed, on a single prototype production line, then spread the fruits of what is learned over several other lines. This again means that, on

efficiency grounds, the production rate may increase over the production life cycle. If, for either reason, production rates increase, then prices decline, if demand does not shift and goods cannot be backordered at the current-delivery price.

2. We did not discount revenues and costs. Therefore, the firm is happy to lose \$1 today if it means even \$1.01 more in profit 5 years hence. In the real world, firms discount cash flows (at least, they do not trade off \$1 today for \$1 in 5 or 10 years). This discounting causes the firm to cut back on production (and losses) early on. Without too much trouble, we can incorporate this into our spreadsheet analysis: Take the spreadsheet with which we began, and instead of adding the quarterly profit figures, sum the discounted profit figures, discounted at whatever quarterly rate is appropriate. Sheet 2 of EXPERIENCE-CURVE computes discounted profits in the final column and then sums. Employing Solver to maximize the sum of discounted profits gives the answer shown in Figure S12.5, where profits are discounted at a rate of 2.5% per quarter. Note that production increases slowly over the 5 years.

	quarter #	quantity	price	revenue	cum prod	cost	cash flow	cash flow discounted
3							quartery discour	2.50%
6					5			
7	1	3293.14	$ 736.55	$ 2,425,559	3298.14	$ 5,261,344	$ (2,835,786)	$ (2,766,620)
8	2	3390.01	$ 728.80	$ 2,470,636	6688.15	$ 3,268,301	$ (797,665)	$ (759,229)
9	3	3454.39	$ 723.65	$ 2,499,766	10142.5	$ 2,797,715	$ (297,949)	$ (276,676)
10	4	3503.27	$ 719.74	$ 2,521,438	13645.8	$ 2,533,252	$ (11,813)	$ (10,702)
11	5	3542.51	$ 716.60	$ 2,538,560	17188.3	$ 2,354,097	$ 184,463	$ 163,038
12	6	3574.98	$ 714.00	$ 2,552,540	20763.3	$ 2,220,701	$ 331,839	$ 286,143
13	7	3602.32	$ 711.81	$ 2,564,183	24365.6	$ 2,115,442	$ 448,741	$ 377,510
14	8	3625.59	$ 709.95	$ 2,574,000	27991.2	$ 2,029,039	$ 544,961	$ 447,275
15	9	3645.53	$ 708.36	$ 2,582,338	31636.7	$ 1,956,049	$ 626,288	$ 501,487
16	10	3662.64	$ 706.99	$ 2,589,444	35299.4	$ 1,893,029	$ 696,416	$ 544,039
17	11	3677.31	$ 705.81	$ 2,595,502	38976.7	$ 1,837,670	$ 757,832	$ 577,578
18	12	3689.85	$ 704.81	$ 2,600,652	42666.5	$ 1,788,360	$ 812,291	$ 603,984
19	13	3700.49	$ 703.96	$ 2,605,001	46367	$ 1,743,929	$ 861,072	$ 624,639
20	14	3709.42	$ 703.25	$ 2,608,635	50076.5	$ 1,703,502	$ 905,133	$ 640,587
21	15	3716.78	$ 702.66	$ 2,611,625	53793.2	$ 1,666,413	$ 945,212	$ 652,637
22	16	3722.71	$ 702.18	$ 2,614,025	57515.9	$ 1,632,139	$ 981,886	$ 661,423
23	17	3727.31	$ 701.82	$ 2,615,882	61243.3	$ 1,600,266	$ 1,015,615	$ 667,457
24	18	3730.65	$ 701.55	$ 2,617,232	64973.9	$ 1,570,460	$ 1,046,771	$ 671,154
25	19	3732.82	$ 701.37	$ 2,618,105	68706.7	$ 1,542,446	$ 1,075,659	$ 672,855
26	20	3733.87	$ 701.29	$ 2,618,528	72440.6	$ 1,515,998	$ 1,102,531	$ 672,842
28							NPV of cash flows	$ 4,951,421

Figure S12.5. The optimal production plan with 2.5% per quarter discounting.

3. An important reason for discounting and for refusing to invest too much in early production is that the product life of many experience-curve

goods is very uncertain. In the example, the firm knows it has 19 quarters to recoup losses it takes in the first quarter. But, in many real-world examples, firms have to worry that "tomorrow" may not come for a particular good. This effectively increases the rate at which they discount profits and losses; firms are less willing to sustain early losses.

4. The analysis assumes that demand does not shift over the 5-year period. In fact, in many cases, demand grows as the product matures, and the firm produces more in later periods simply because demand is greater at any given price. Note, however, that while this may mean that production levels should increase, it does not mean that prices should fall. In fact, insofar as the firm can affect the size of the market it faces later by building a customer base, it has increased incentives to price low early on. On the other hand, as we saw in Chapter 7, it may be possible to use slowly declining prices as a way to discriminate among customers.

5. The model assumes that a single firm faces consumers. In real-world applications, the firm must be concerned with the actions of competitors. The impact of competition is quite complex and can push the "solution" we obtained in either direction. For example, if a firm has an early lead in a particular product and there is little or no spillover from one firm to another in terms of experience, then the leading firm may price very aggressively (that means, low prices) early on to forestall the entry of competitors. The large airframe manufacturers are sometimes seen as acting in this fashion; Boeing priced 747s very aggressively and captured the market for large-jumbo planes for a long period of time, leaving McDonnell-Douglas, Lockheed, and later Airbus to compete in the medium-jumbo segment. On the other hand, when competition is hard to restrain or there is spillover from one firm to another of the fruits of experience, firms are less inclined to invest in knowledge, because knowledge is correspondingly less proprietary. This results in pricing strategies that look more like pricing down the experience curve.

We can fairly easily incorporate effects 1 and 2 into the model and, with some heroic assumptions, incorporate effect 3. But, to handle 4 and 5, we need more tools than we have at our disposal.