

# Chapter 13 Material

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The solution to Problem 13.1 follows immediately. Following that, the model for the GM Truck Coupons assignment is developed.

**13.1** (a) If consumers pay precisely what producers receive, then  $p = q$ , and equilibrium is found where supply equals demand. Algebraically, this is where

$$S(p) = 1000(p - 4) = 2000(10 - p) = D(p),$$

where I substitute  $p$  for  $q$  in the demand side of this equation, since  $p = q$ . Dividing both sides of the equation by 1000, this gives

$$p - 4 = 2(10 - p) \quad \text{or} \quad p - 4 = 20 - 2p \quad \text{or} \quad 3p = 24 \quad \text{or} \quad p = 8,$$

at which point the quantity supplied and demanded is 4000 units. Figure S13.1 shows this in a picture.

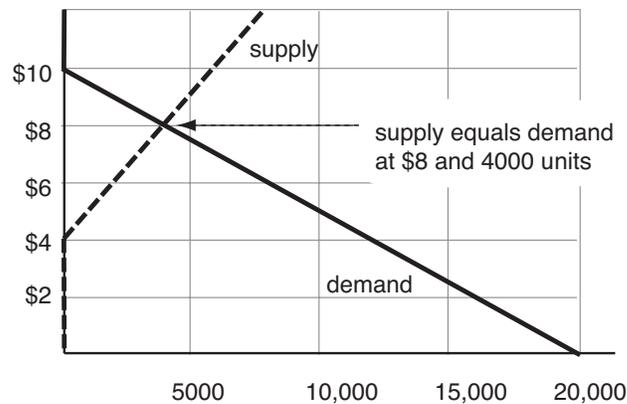


Figure S13.1 Supply equals demand in Problem 13.1.

(b) Let  $P$  denote the price on the price tag of the good. With a sales tax paid by the buyer, the “supplier price,” or what the producer gets from the sale of a unit, is  $P$ . And the “buyer price,” or the net amount paid by a buyer, is  $P$  plus the tax. So, in this part of the problem,

$$p = P \quad \text{and} \quad q = 1.1P.$$

Hence, in terms of the price-tag price  $P$ , supply equals demand where

$$S(P) = 1000(P - 4) = 2000(10 - 1.1P) = D(1.1P),$$

which is  $P = \$7.50$ . The quantity supplied and demanded is 3500 units, and government revenue from the sales tax is  $3500 \times \$0.75 = \$2625$ .

(c) Keeping  $P$  as the price-tag price of the good, in this case, the consumer pays  $P$ , so  $q = P$ , but sellers of the good take in only  $0.9P$  (with 10% going to the government). Therefore, supply equals demand in this case at

$$S(0.9P) = 1000(0.9P - 4) = 2000(10 - P) = D(P),$$

which is  $P = \$8.276$ , giving a quantity bought and sold of 3448 units and government tax revenues of  $3448 \times \$0.8276 = \$2853.79$ .

It is worth noting that the answers in parts b and c are different, because as far as a “tax” goes, the tax in part b is less severe than the tax in part c: The size of a tax is the gap between what the buyer pays and what the seller gets. For this sort of sales tax, look at the ratio between the two. In part b, the ratio is 1.1:1. In part c, it is 1:0.9. And

$$\frac{1}{0.9} = \frac{1.111\dots}{1} \neq \frac{1.1}{1}.$$

Suppose, though, in part c we said that for every \$1 received, sellers had to remit \$0.090909... to the government, keeping \$0.909090... for themselves. The point is that

$$\frac{1}{0.9090\dots} = \frac{1.1}{1}.$$

Then if  $P$  is the price-tag price of the good,  $P = q$  is what buyers pay, while  $p = 0.9090\dots \times P$  is what sellers get. So supply equals demand where

$$1000(0.90909\dots \times P - 4) = 2000(10 - P),$$

which is  $0.909090\dots \times P - 4 = 20 - 2P$ , or  $2.90909\dots \times P = 24$ , or

$$P = \frac{24}{2.90909\dots} = 8.25.$$

Recalling that this is the price the consumers pay, they demand  $2000(10 - 8.25) = 2000 \times 1.75 = 3500$ , while producers receive  $0.9090\dots \times 8.25 = 7.50$  at which price they supply 3500. The answer is the same as in part b.

Moral: A tax imposed on buyers gives the same outcome as the “same” tax imposed on sellers. But you have to be careful to make sure the taxes in the two regimes are really the same.

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Now for the GM Truck Coupon Model.

The key to modeling this situation is to recognize that there are *two* markets with which you must be concerned. The first, and obvious, market is the market in GM light trucks, with GM setting the price as it has market power. But there is (potentially) a second market, the market in unused GM Truck coupons, with many sellers—holders of coupons who choose not to buy a GM light truck and so would be happy to unload their coupon in order to make a bit of money—and many buyers, people who want to buy a GM light truck and would be happy to have a coupon that is worth some amount of money from GM. Since there are many buyers and many sellers, and coupons are “commodity items”—any one is as good as any other—we just need to pass the test of full information to conclude that supply-equals-demand is the model to use to estimate the price of coupons in their market.

Chapter 13 suggests that you need to think whether someone will “make” the market. I think it is obvious that, in this case, someone will do this. If no one else steps up, it is clearly in the interest of GM dealers to make this market: A customer with a coupon is a customer whose real “price” for a truck is \$1000 to \$500 less than what the dealer gets, since it is GM back in Detroit and not the dealer who pays for a redeemed coupon. If a customer and dealer could settle on a price  $P$  for a sale without a coupon, the dealer ought to be able to get  $P + 500$ , more or less, if he (the dealer) can hand a coupon to the customer and say, “Make a deal with me today, and I’ll write

your name in on this coupon and (even) advance you the \$500 that you'll get from GM." (My guess is that, to save on paperwork, GM will probably arrange with dealers that they—the dealers—can collect the coupons and redeem them in bulk.)

So, with this in mind, I'm going to assume in my model that a competitive market in unused coupons is set up, in which some price  $p$  for the coupons will be established. And I will apply supply-equals-demand to try to figure out what  $p$  should be.

Let's begin with the original terms of the proposed deal: Coupons are worth \$1000 off the purchase of a new light truck if used by the original bearer and worth \$500 off if used by a third-party buyer, for the next 15 months. I think it is pretty clear that no one would buy a coupon if  $p$  exceeded \$500, so demand above \$500 will be zero, and supply can only equal demand at prices  $p$  below \$500.

But as long as  $p$  is below \$500, everyone who wants to buy a GM light truck in the 15 month window will want to buy with a coupon. That number depends, of course, on the price  $P$  that GM sets for its light trucks, and I'm going to assume (at least for this simple model) that GM sets a price  $P$  and sticks to it over the 15 month period. If that is so, the effective price of a truck for a third-party buyer isn't  $P$  but instead is  $P - 500 + p$ . And how much demand is there for GM light trucks from third-party buyers? The case-let said that overall demand for GM light trucks over the next 15 months at an effective price of \$20,000 was for 2 million trucks, with an elasticity of  $-4$ , that 70% of those sales came from third-party customers (those who wouldn't have gotten a coupon in the mail), and that the elasticity of the third-party buyers was the same  $-4$ .

The case-let suggests using a linear demand function, one of the form  $D(P) = A - BP$  for constants  $A$  and  $B$ . If we look at the demand for third-party buyers first, we know that  $A$  and  $B$  satisfy

$$1,400,000 = A - 20,000B.$$

We also know that elasticity is  $D'(P)P/x$ , so we know that

$$\frac{-B \times 20,000}{1,400,000} = -4 \quad \text{or} \quad B/70 = 4, \quad \text{so} \quad B = 280.$$

If you do the algebra, this gives  $A = 7$  million.

So if GM sets the price of its light trucks at  $P$  and the price of coupons in the coupon market is  $p$ , demand for GM truck coupons will be

$$7 \text{ million} - 280(P + p - 500),$$

that being the number of third-party buyers who wish to purchase a GM light truck under these conditions. That number is largest when  $P$  and  $p$  are as small as possible: Of course,  $p$  can't be less than zero, and I don't think it is reasonable to suppose that GM will price its light trucks at a price such that GM nets less than its marginal cost of \$15,000, so a lower bound on  $P$  is \$15,500, and an upper bound on demand from third-party buyers is

$$7 \text{ million} - 280(15,500 - 500) = 2,800,000.$$

And what about supply? Any one of the 4.7 million folks who gets a coupon has a choice—buy a GM light truck with the coupon, or sell the coupon for the price  $p$ . So how many of the folks who hold coupons are willing to buy a GM light truck and use their coupon, rather than selling it?

I need their demand function for GM light trucks, knowing that at the price \$20,000, they will buy 600,000 trucks over the 15-month period, and knowing that their elasticity of demand is  $-4$  at \$20,000. Assuming their demand is linear, taking the form  $D_o(P) = A_o - B_oP$ , where the subscript o's are for "old truck owners," we have

$$600,000 = A_o - 20,000B_o \quad \text{and} \quad -4 = \frac{-B_o \times 20,000}{600,000}$$

which can be solved giving

$$A_o = 3 \text{ million} \quad \text{and} \quad B_o = 120.$$

So even if GM sets the price of its light trucks at \$15,500 (and I think it is extremely unlikely that they'll go this low), so that the old car owners' effective price is \$14,500, and even if old car owners take no notice of the money they could get ( $p$ ) by selling their coupons, the largest number of light trucks they would buy is 3 million less  $120 \times 14,500 = 1,260,000$ . *This means that of the 4.7 million coupons initially mailed out, **at least** 4.7 million less 1.26 million = 3.44 million coupons will be supplied to the unused-by-their-initial-owners coupon market.*

And note, all of these estimates are very weak—if we set the lowest price that GM might set for its trucks at \$18,000—which would be less than it set before, when there were no coupons—we’d get an upper bound on demand for coupons of 2.1 million and a lower bound of supply of 3.6 million, even if the coupons were free ( $p = 0$ ).

So where does supply equal demand in the unused coupon market? As  $p$  increases, you decrease demand a bit, and you increase supply (the old truck owners who are on the margin of buying a GM light truck with their coupon think it makes more sense to sell the coupon for  $p$  and, say, buy a Toyota). So the only price where supply equals demand is at  $p = 0$ . Now, in reality, I doubt that  $p = 0$  will do. Whoever is making this market will need to get old-truck owners who don’t plan to use their coupon to take the time and effort to give them up. And, early in the 15 month period, an old-truck owner may want to hold on to her coupon, just in case she discovers later in the 15-month period that she does indeed want to buy a GM light truck. But I’d bet that if dealers offered old truck owners, say, \$50 for their coupons, they’d be flooded with coupons, which they could then unload to third-party buyers who are about to buy a GM light truck.<sup>1</sup>

This provides the first conclusion at which the judge arrived when facing with the initial settlement proposal. *For the majority of the old truck owners—those who didn’t want to buy a new light truck—the coupons would command very little in the unused-coupon market, so unless an old-truck owner was willing to buy a new truck, she gets little out of the settlement.*

But what about the old-truck owners who want to buy a new light truck? Wouldn’t this be worth \$1000 apiece to them? It would, if GM kept the posted price of its light trucks where they were—in the model, at \$20,000. But GM is unlikely to do that.

Assuming that  $p = 0$  (more or less) and GM posts a price of  $P$ , its total sales will be

$$7,000,000 - 280(P - 500) + 3,000,000 - 120(P - 1000),$$

because with  $p = 0$ , third-party buyers face an effective price of  $P - 500$  and old truck owners face  $P - 1000$ . Do the algebra, and this simplifies to

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<sup>1</sup> Students sometimes ask, “Why not *sell* the coupons to the prospective buyers, since the coupons are worth \$500 to them? Sure, the dealers can do this. But as they bargain with the customer, the bargain they strike will reflect the price they charge for the coupon, on almost a dollar-for-dollar basis. Sales tax on the vehicle could complicate the story a bit. But, aside from that, the dealers might as well give away the coupons and “make it up” in terms of the price they negotiate with the customer.

$10,260,000 - 400P$ . So inverse demand is

$$P(x) = \frac{10,260,000}{400} - \frac{x}{400} = 25,650 - \frac{x}{400},$$

and marginal revenue is  $25,650 - x/200$ . I'd like to set  $MC = MR$ , but there is a problem: marginal cost is different for the two types of buyer. For third-party buyers, it costs \$15,500 per car sold, while for old truck owners, it costs \$16,000.

I have a choice of strategies for finding the optimal value of  $P$ . Perhaps the easiest is to do it in a spreadsheet: In Table S13.1, you find a simple spreadsheet that I created for this purpose. The first row is the driving variable  $P$ , the price GM posts. Assuming  $p = 0$ , you have the effective price for both groups and their demands. Total sales units comes next, together with total gross revenues. Total production costs are just \$15,000 times the number of units sold. And givebacks for coupon redemption is \$1000 times the number of sales to old truck owners plus \$500 times the number of third-party buyers.

Posted price $P$	\$20,000	\$21,000	\$20,650
Effective price for old truck owners	\$19,000	\$20,000	\$19,650
Demand by old truck owners	720,000	600,000	642,000
Effective price for third-party buyers	\$19,500	\$20,500	\$20,150
Demand by third-party buyers	1,540,000	1,260,000	1,358,000
Total sales (units)	2,260,000	1,860,000	2,000,000
Total revenue	\$45,200,000,000	\$39,060,000,000	\$41,300,000,000
Total production costs	\$33,900,000,000	\$27,900,000,000	\$30,000,000,000
Total givebacks for coupon redemption	\$1,490,000,000	\$1,230,000,000	\$1,321,000,000
NET PROFIT	\$9,810,000,000	\$9,930,000,000	\$9,979,000,000

Table S13.1. Finding the optimal  $P$  for GM to set.

The table has three columns. The first explores what happens if GM keeps  $P$  at \$20,000. The net profit should be benchmarked against the \$10 billion GM makes with no coupons (2 million vehicles times \$5,000 margin on each); if GM keeps  $P$  at \$20,000, this will cost it \$190 million. But why not raise  $P$ ? The second column tries  $P = \$21,000$ , which lowers the cost of the program to \$70 million. And, using Solver or just hunt-and-peck, you'll find that the

best price to charge is  $P = \$20,650$ , so that the program costs GM only \$21 million. And of the 4.7 million old truck owners, only 642,000 buy a new light truck, at an effective price of \$350 less than what they would have paid had there been no program and no coupons. (For 600,000, the “value” of the program is \$350. For the remaining 42,000, it is less, since they were only induced to buy a GM light truck because its effective prices to them was reduced.)

*Conclusion: The judge was certainly correct. For the old-truck owners, whether they want to buy a new truck or (especially) if not, this planned settlement is not a very good deal. And, insofar as the point of the settlement is to punish GM, it isn't doing the job at all.*

(Read the following paragraph only if the math isn't bothering you too much.) You can also solve this problem without Excel, but it is harder. In particular, because  $P$  has to be the same for both groups, you need to optimize in  $P$  and not in quantities. Here's how: For a given  $P$ , GM sells  $7,000,000 - 280(P - 500)$  cars to third party buyers. Its net from each sale is  $P - 500 - 15,000$ , so this gives a profit contribution of

$$[7,000,000 - 280(P - 500)] \times [P - 15,500] = \\ - 280P^2 + (280)(15,500)P + (280)(500)P + 7,000,000P + \text{constant.}$$

To old car buyers, it sells  $3,000,000 - 120(P - 1000)$  cars, netting from each  $P - 1000 - 15,000$ , giving a contribution of

$$[3,000,000 - 120(P - 1000)] \times [P - 16,000] = \\ - 120P^2 + (120)(16,000)P + (120)(1000)P + 3,000,000P + \text{constant.}$$

GM wants to choose  $P$  to maximize the sum of these two, and ignoring the constants (since they are constant), it wants to choose  $P$  to maximize

$$-400P^2 + [(280)(15,500) + (280)(500) + (120)(16,000) + (120)(1000) + 10,000,000] P.$$

Taking the derivative and setting it equal to zero, the optimal value of  $P$  is

$$P = \frac{10,000,000 + (280)(15,500) + (280)(500) + (120)(16,000) + (120)(1000)}{800} = 20,650.$$

(I told you it was easier with the spreadsheet.)



Now to add some elaboration. In Figure S13.2, you see a spreadsheet that I created from the spreadsheet used to create Table S13.1. It adds two features:

- It allows me to enter a number for the elasticity of demand of the old-truck owners different from  $-4$  and, using the usual formula, it computes the corresponding elasticity of the third-party buyers consistent with an overall elasticity of  $-4$ .
- It allows me to enter two “prices” for a coupon. The amount received by an old-truck owner if she sells her coupon on the unused-coupon market, and the price paid by third-party buyer for such a coupon. I allow a “split” between the two on grounds that market-makers in the unused-coupon market will, presumably, want something for making this market; the difference between the selling and buying price is that something.

	B	C	D	E	F	G
1						
2		<b>GM TRUCK COUPON SPREADSHEET</b>				
3						
4		elasticity of old-truck owners	-4	-4.5	-4	-4
5		elasticity of third parties (computed)	-4.00	-3.79	-4	-4
6						
7		price received by old-truck owner for unused coupon	\$0.00	\$0.00	\$200.00	\$200.00
8		price paid for coupon by third-party buyer	\$0.00	\$0.00	\$200.00	\$250.00
9						
10		price GM sets	\$20,650.00	\$20,668.75	\$20,550.00	\$20,532.50
11		effective price old-truck owners	\$19,650.00	\$19,668.75	\$19,750.00	\$19,732.50
12		effective price third-party buyers	\$20,150.00	\$20,168.75	\$20,250.00	\$20,282.50
13		% change price for old-truck owners	-1.75%	-1.66%	-1.25%	-1.34%
14		% change price for third-party buyers	0.75%	0.84%	1.25%	1.41%
15						
16		% change in demand by old-truck owners	7.00%	7.45%	5.00%	5.35%
17		% change in demand by third-parties	-3.00%	-3.19%	-5.00%	-5.65%
18						
19		demand by old-truck owners	642000.001	644718.7501	630000.0011	632100.0037
20		demand by third parties	1358000.002	1355281.25	1330000.003	1320900.009
21						
22		total cars sold	2000000.003	2000000	1960000.004	1953000.012
23						
24		revenue from old-truck owners	\$12,615,300,015	\$12,680,811,916	\$12,316,500,016	\$12,346,493,303
25		revenue from third-party buyers	\$27,363,700,036	\$27,334,328,715	\$26,666,500,040	\$26,460,929,384
26		total cost	\$30,000,000,051	\$30,000,000,006	\$29,400,000,056	\$29,295,000,187
27		profit contribution	\$9,979,000,000	\$10,015,140,625	\$9,583,000,000	\$9,512,422,500
28						
29		baseline profit	\$10,000,000,000	\$10,000,000,000	\$10,000,000,000	\$10,000,000,000
30						
31		COST TO GM OF PROGRAM (\$ millions)	\$21	-\$15	\$417	\$488

Figure S13.2. Variations on the basic GM-truck coupon model.

Each column in the spreadsheet represents a different scenario: Column D is the base-case analyzed already; in Column E, we keep the price of the

coupon at \$0 but change the elasticity of demand of the old-truck owners so that they are *more* elastic; in Column F we go back to the original elasticities but put the buying and selling price of a coupon at \$200; and in Column G we put a gap of \$50 between the buying and selling price of a coupon. In each column, Solver was used to optimize (from GM's perspective) the price they set (the entry in row 10), where the objective is to make the cost of the program to GM (row 31) as small as possible.

Please look carefully at the numbers in rows 11 and 12, the effective prices for the two sorts of buyers. It is assumed that the coupon is worth \$1000 off the list for the old-truck owners. But if you look at cell F11, you'll see that the effective price for old-truck owners is only \$800 less than the list price. This is because, if the old-truck owners can get \$200 for a coupon, using the coupon for their own purchase has an opportunity cost to them of \$200, which (effectively) raises the price they pay for the truck they buy by this opportunity cost. As for the third-party buyers, it is assumed that the coupon gives them a \$500 discount, the effective price of a new truck for them is the list price, less this discount, plus the price of a coupon. So, for example, in column G, where the price of a coupon to them is \$250, the effective price to them is only \$250 less than the list price of \$20,532.50.

Now go down to Row 31, to see the results for GM of these different scenarios. The base-line case, Column D, costs GM \$21 million, as we saw back in Table C13.1. In Column E, where we assume that old-truck owners have more elastic demand, the "cost" of the program is that GM makes \$15 million over the no-coupon situation. This is price discrimination at work. Since old-truck owners have more elastic demand than third-party buyers, GM would like to charge the old-truck owners a lower price. The coupons enable this.

And in Columns F and G, we see the impact of nonzero prices for the unused coupons. The (optimized-for-list-price) cost to GM of the coupon program has sky-rocketed, to \$417 million in the Column F scenario and \$488 in the Column G scenario.

Why? Think of it this way. Suppose the government came along and fined GM \$200 for each new light truck it sold, but didn't give that money to buyers of the new light truck. This would cost GM (and the customers) substantial amounts of money—someone has to pay for all that money flooding into the government treasury. Will cost be borne more by GM or by its customers? We'll answer that question systematically in Chapter 16 when we consider the impact of taxes on goods but, for now, trust me when I assert that, roughly, GM in this situation would sustain a loss in profit roughly equal to the amount of revenue the government collects. (If you want to look ahead

in the text to see where this comes from, look at page 361.) With around 2 million cars sold, that's roughly \$400 million.

The point is that while GM isn't paying an actual "fine," the effect of these coupons is the same (in Column F): GM is effectively receiving \$19,550 for each new truck it sells to an old-truck owner—the list price of \$20,550 less the rebate of \$1000—but for an old-truck owner, the effective price is \$19,750. And for third-party buyers, the effective revenue per truck for GM is \$20,050, but the effective price to the buyer is \$20,250. When the coupons in the unused-coupon market have a significant price (in that market), that price works roughly as would a fine imposed on GM, and *that* is quite costly for GM.

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So why is this proposed settlement so bad (for the public, and good for GM), while the second proposed settlement was okay for the public?

The key is supply-equals-demand in the unused-coupon market. In this, the first settlement, there were "too many" coupons for uses for the coupons. So in the unused-coupon market, the price of unused coupons would be close to zero. A relative handful of the old truck owners would benefit from lower prices, although a profit-maximizing GM would shave that benefit from the nominal \$1000 to a mere \$350 or so. GM would lose a token amount of money, but only because it is forced into "price discrimination" that it doesn't wish to do. It is forced to offer a lower effective price to old truck owners than to third party buyers, and as long as their elasticities of demand are equal, this is not profit improving.

(But if the old-truck users had more elastic demand than did third-party buyers, GM might actually make money out of this settlement. Although in case the elasticities were different in that direction, it is hard to see why GM wouldn't already be price-discriminating, using something like a "loyalty discount" for previous owners.)

But in the second settlement, while the number of coupons was increased to 5.8 million coupons, the coupons retained a face value of at least \$250 for 33 months, over which time GM expected to sell over 10 million vehicles. So now demand for coupons far outstrips supply, and supply-equals-demand in the unused-coupon market ensures that the price  $p$  in the unused-coupon market will be considerable. (It is a tougher exercise in modeling, so I won't take you through it, but if we ignore the time value of money,  $p$  will be set by the marginal use to which a coupon is put, which is a \$250 reduction in price for a third party buyer. That is, economic theory would predict  $p = \$250$ .)

And if unused coupons have a nontrivial price, GM loses a considerable amount of money: The back-of-the-envelope estimate in this case (where GM is the monopoly seller of its vehicles and its marginal cost of production is constant—see Chapter 16) is the market price of the coupon times the number of vehicles it sells.

Please note: the point of the program is not necessarily to harm GM but instead to provide value to old-truck owners. From that perspective, it is the market price in the unused coupon market that is their main compensation. Some old-truck owners get some additional compensation, if they are willing to buy a new GM vehicle. But, from the perspective of the old-truck owners as a class, each one can get at least the market price of an unused coupon out of this deal, which (the theory predicts) will be in the neighborhood of \$250.

Of course, the Certificate Redemption Group stepped forward, willing to buy coupons for \$100 apiece, saying that they expected to be able to sell them for \$150 to \$200 apiece. Frankly, this is a steal; old-truck owners should hold out for a better price for their unused coupons. I've searched the web for what happened next, and I can't find anything (except for squabbling between GM, the Texas class-action suit judge, and a Louisiana judge who was dealing with the same issue in a separate class-action suit, concerning whether CRG should be allowed to make this offer). Eventually CRG was allowed to proceed. But be that as it may, if I'm a GM dealer, I'm going to contact every certificate owner I can find and offer them at least \$125 (and old-truck owners should wait for that number to rise, as well). I would expect (but I don't know whether) the power of supply-equals-demand would force CRG, facing competition, to up its offer to closer to \$250.

Finally, you were asked, "What if, in the second program, GM picked 11.6 million names randomly out of the phone book and mailed to each of those folks an identical coupon?" I hope you now see: This will put the unused-coupon market into a condition of much more supply than demand, pushing down (to nearly zero) the market price of an unused coupon, drastically reducing the benefit of the program to old-truck buyers and, not coincidentally, reducing the cost to GM of the whole program dramatically.