

# Chapter 14 Material

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Solutions to problems from Chapter 14 are presented. Then, two “bonus” topics are covered: A toy model of industry-wide price effects; and a discussion of monopolistic competition.

**14.1** The \$1 million fixed cost is irrelevant, since it cannot be avoided. The firm’s “relevant” total cost function is  $\$4,000,000 + 5x + x^2/10,000$ , and the firm will not supply at any price  $p$  that is less than the minimum average cost generated by this “relevant” cost function. To find minimum average cost, equate marginal and average costs, or

$$\frac{4,000,000}{x} + 5 + \frac{x}{10,000} = 5 + \frac{2x}{10,000} \quad \text{or} \quad \frac{4,000,000}{x} = \frac{x}{10,000}'$$

which gives  $x^2 = 40$  billion or  $x = 200,000$ , for a minimum “relevant” average cost of \$45. At prices above \$45, the firm supplies along its marginal cost function, or  $p = 5 + 2x/10,000$ , or  $x = 5000(p - 5)$ . Therefore the overall supply function is

$$s(p) = \begin{cases} 0, & \text{if } p < \$45, \\ 0 \text{ or } 200,000, & \text{if } p = \$45, \text{ and} \\ 5000(p - 5), & \text{if } p > \$45. \end{cases}$$

**14.2** Efficient scale is where average cost equals marginal cost.  $AC = MC$  here is

$$\frac{F_1 + F_2}{x} + 3 + \frac{x}{40,000} = 3 + \frac{x}{20,000} \quad \text{or} \quad \frac{F_1 + F_2}{x} = \frac{x}{40,000},$$

or  $x = \sqrt{40,000(F_1 + F_2)} = 200\sqrt{F_1 + F_2}$ . The problem tells us that this is  $x = 60,000$ , so

$$60,000 = 200\sqrt{F_1 + F_2} \quad \text{or} \quad 300 = \sqrt{F_1 + F_2} \quad \text{or} \quad F_1 + F_2 = \$90,000.$$

The firm supplies positive levels of output at all prices above \$5, which means that when price hits \$5, the firm's gross profits, gross of fixed costs, just covers the fixed cost  $F_2$ . Since it is competitive, it will produce along its marginal-cost function where it produces at all, so its supply at \$5 is the solution to  $5 = 3 + x/20,000$  or  $x = 40,000$ . At this level of production and a \$5 price, its revenues are \$200,000. Its costs gross of fixed costs are

$$3 \times 40,000 + \frac{40,000^2}{40,000} = \$160,000,$$

so the difference, or \$40,000, is  $F_2$ . And then from  $F_1 + F_2 = \$90,000$ , we know that  $F_1 = \$50,000$ .

**14.3** With or without a fixed cost, this firm, being competitive, supplies nothing at prices below its minimum "relevant" average cost, where relevance here means ignoring all unavoidable fixed costs. At minimum average cost, it supplies 0 or efficient scale. At prices above minimum average cost, it produces along the *increasing* portion of its marginal cost function (really, at levels beyond efficient scale), which are solutions to

$$p = 8 - \frac{x}{10} + \frac{x^2}{2000}.$$

Solving for  $x$  in this quadratic equation and taking the greater root to be on the increasing part of marginal cost gives

$$\begin{aligned} s(p) &= \frac{0.1 + \sqrt{0.01 - 4 \times (8 - p) \times 0.0005}}{2 \times 0.0005} \\ &= 1000 \times [0.1 + 0.1\sqrt{1 + 0.2(p - 8)}] = 100[1 + \sqrt{1 + 0.2(p - 8)}]. \end{aligned}$$

Hence, in both the cases of no fixed cost and completely avoidable fixed cost,

$$s(p) = \begin{cases} 0, & \text{if } p < \min \text{ AC,} \\ 0 \text{ or efficient scale,} & \text{if } p = \min \text{ AC, and} \\ 100[1 + \sqrt{1 + 0.2(p - 8)}], & \text{if } p > \min \text{ AC.} \end{cases}$$

The final tasks are to find the minimum average cost and efficient scale for the two cases.

To do this, we equate marginal cost to average cost. If there is no fixed cost, we have to solve  $8 - x/10 + x^2/2000 = 8 - x/20 + x^2/6000$ , which is  $x/20 = 2x^2/6000$ , or  $3000/20 = 150 = x$ . That is, efficient scale in this case is 150. And to find min AC, we plug efficient scale back into the marginal cost function, getting  $8 - 15 + 22500/2000 = \$4.25$ .

If there is a fixed cost of \$10,000, we instead have to solve  $8 - x/10 + x^2/2000 = 10,000/x + 8 - x/20 + x^2/6000$ , which is a cubic equation, beyond my powers to do analytically. So I went to Solver and asked it to find the value of  $x$  that minimized average cost, and it returned: efficient scale = 369.6; and min AC = \$39.343.

**14.4** A change of variables will help clarify. I let  $z$  be the “net trade” of this consumer in the good in question. If  $z$  is negative, the consumer is a net seller of the good. If  $z$  is positive, she is a net buyer. Since she has an endowment of 100 units of the good,  $z$  is constrained to be no smaller than  $-100$ ; that is, she can’t sell what she doesn’t have.

If  $z$  is her net trade, then her consumption of the good is  $x = 100 + z$ . And, therefore, her utility function written in terms of  $z$  is  $10 \ln(100 + z + 1) + m$ . So her problem is to maximize (over  $z$ )  $10 \ln(100 + z + 1) + m$  subject to three constraints:  $m = 1000 - pz$ , where  $p$  is the price of the good;  $z \geq -100$ ; and  $pz \leq 1000$  (she can’t borrow money to finance purchase of the good). Replace  $m$  with  $1000 - pz$  in the objective function to get  $10 \ln(101 + z) + 1000 - pz$ , and taking the derivative in  $z$  and setting it equal to zero gives

$$\frac{10}{101 + z} = p \quad \text{or} \quad 10 = 101p + pz \quad \text{or} \quad z = \frac{10 - 101p}{p}.$$

This gives the consumer’s supply-demand function—if it is negative, she is supplying; if positive, demanding—as long as the constraints  $pz \leq 1000$  and  $z \geq -100$  are not violated. As for the first of these constraints  $pz = 10 - 101p$ , so as long as  $p \geq 0$ ,  $pz \leq 10$ ; so the constraint  $pz \leq 1000$  is never a problem. But as  $p$  approaches  $+\infty$ , the expression we have for  $z$  as a function of  $p$  approaches  $-101$ . That is, as the price of the good gets very, very large, the consumer wishes to sell 101 units. So we amend the demand function as follows: For  $p$  such that

$$\frac{10 - 101p}{p} < -100, \quad \text{or} \quad 10 - 101p < -100p \quad \text{or} \quad 10 < p,$$

the consumer sells all her 100 units. This gives her supply-demand function as

$$z(p) = \begin{cases} (10 - 101p)/p, & \text{if } p \leq 10, \text{ and} \\ -100, & \text{if } p > 10. \end{cases}$$

Note that  $p = 10/101$  is the critical price at which she turns from demander (if  $p < 10/101$ ) to supplier (if  $p > 10/101$ ).

**14.5** If  $TC(x) = 4x + x^2/2$ ,  $MC(x) = 4 + x$ . This is an increasing function and the total cost function has no fixed costs, so the individual firm's supply function is this: At prices below 4, supply nothing; at prices 4 and above, supply  $s(p)$  that solves  $4 + s(p) = p$ , or  $s(p) = p - 4$ . If 10 identical firms are in the industry, then industrywide supply is 10 times the supply from any single firm, which is

$$S(p) = \begin{cases} 0, & \text{if } p < 4, \text{ and} \\ 10p - 40, & \text{if } p \geq 4. \end{cases}$$

Equilibrium is where supply equals demand. For the demand function  $D(p) = 10(20 - p)$ , at  $p = 4$ , demand is 160, so the intersection must take place at a price larger than 4, and we can find that equilibrium price by setting

$$D(p) = 10(20 - p) = 10p - 40 = S(p)$$

which gives

$$240 = 20p \quad \text{or} \quad p = 12.$$

At this price, industrywide demand equals industrywide supply equals 80 units, so each firm supplies 8 units. This gives each firm revenue of  $12 \times 8 = 96$ , costs of  $4 \times 8 + 8^2/2 = 32 + 32 = 64$ , and a profit of  $96 - 64 = 32$ .

**14.6** (a) If there is no possibility of entry or exit, we can ignore the fixed cost. Marginal cost for any one of the five firms is  $MC(x) = 2 + x/50,000$ , so at the price  $p$ , a firm supplies the solution to marginal cost equals price, or

$$s(p) = 50,000(p - 2).$$

Of course, this is for prices  $p$  above \$2 only; for lower prices, each firm supplies zero.

Since there are five such firms, industry supply is five times this, or  $S(p) = 250,000(p - 2)$ . So market equilibrium is where supply equals demand, or

$$250,000(p - 2) = 500,000(42 - p) \quad \text{or} \quad p - 2 = 2(42 - p) \quad \text{or} \quad 3p = 86,$$

which is  $p = \$28.67$ .

(b) If there is free entry and exit, long-run equilibrium will occur where active firms are just covering their fixed costs, which means that price will be at the firms' minimum average cost. We find this in the usual way, equating average cost to marginal cost, to get

$$\frac{10 \text{ million}}{x} = \frac{x}{100,000} \quad \text{or} \quad x^2 = 1,000,000,000 \quad \text{or} \quad x = 1 \text{ million.}$$

This is efficient scale: Minimum average cost is obtained by plugging this quantity into marginal cost (or average cost), to get minimum average cost of \$22. Now at that price, total market demand is 10 million, so the long-run equilibrium has 10 active firms, each producing 1 million units, with an equilibrium price of \$22 and 0 profit for each active firm.

**14.7** (a) Since the firms are identical and there is unlimited possibility of entry (and cost = 0 if a firm exits), in the long-run equilibrium, no firm can make a profit or take a loss. Therefore, the long-run equilibrium price must be the minimum average price of any firm, and firms that produce a positive amount must be producing at efficient scale. Total costs are given by

$$TC(x) = 100 + 3x + 0.04x^2,$$

and therefore average costs are

$$AC(x) = \frac{100}{x} + 3 + 0.04x.$$

To find minimum average cost and efficient scale, I set  $AC = MC$  getting

$$\frac{100}{x} + 3 + 0.04x = 3 + 0.08x \quad \text{or} \quad \frac{100}{0.04} = x^2 \quad \text{or} \quad x = \frac{10}{0.2} = 50.$$

So  $x = 50$  is the efficient scale. And the minimum level of average cost is

$$MC(50) = 3 + 0.08 \times 50 = 3 + 4 = 7.$$

So, in a long-run equilibrium, the price must be  $p = 7$ . At this price, the total demand is

$$D(7) = 200(10 - 7) = 600,$$

which therefore must equal total supply. Since each firm (that is not producing 0) must produce 50 units at  $p = 7$ , this means that there must be 12 active firms. Of course, each of these active firms makes a profit of 0.

(b) To do the rest of this problem in a way that promotes understanding of the structure of these models, look at the table in Table S14.1. The four columns describe the situation for the status quo, the new short-run equilibrium, the new intermediate-run equilibrium, and the new long-run equilibrium. Panel a has the answers for the status quo, taken from part a.

Now ask yourself, which cells in the other three columns can you fill in with no computation whatsoever? Panel b shows these “automatic” entries; please review why these entries are all “automatic.”

Of course, this does not finish the problem. Panel c shows the rest of the table, based on the arguments that follow.

Demand suddenly changes to  $D(p) = 200(12 - p)$ . If, in the short run, firms cannot change their production decisions, then each of the 12 active firms continues to produce 50 units, for a totally inelastic (vertical) supply of 600 units. Price must adjust so that demand is 600 units, or

$$D(p_{SR}) = 200(12 - p_{SR}) = 600,$$

which gives  $p_{SR} = 9$ . At this price, each of the 12 firms has a profit margin of 2 per unit produced, and since they produce 50 units each, this gives each firm a profit of 100.

In the intermediate-run, the firms supply along their marginal cost curve. (Even though they have fixed costs, they cannot exit and avoid those fixed costs, so the fixed costs are irrelevant.) Each firm’s marginal cost function is

	status quo	new short run	new intermediate run	new long run
price	\$7			
total quantity	600			
number of firms	12			
quantity per firm	50			
profit per firm	\$0			

(a) The data from part a: the status quo.

	status quo	new short run	new intermediate run	new long run
price	\$7			\$7
total quantity	600	600		
number of firms	12	12	12	
quantity per firm	50	50		50
profit per firm	\$0			\$0

(b) Easy answers for part b.

	status quo	new short run	new intermediate run	new long run
price	\$7	\$9	\$8.14	\$7
total quantity	600	600	771.43	1000
number of firms	12	12	12	20
quantity per firm	50	50	64.29	50
profit per firm	\$0	\$100	\$65.27	\$0

(c) The full answer for part b.

Table S14.1. Problem 14.7: Solving in stages. After finding the status-quo position and filling in the values into the first column, a number of entries for the new short-run, intermediate-run, and long-run equilibria are quite simple. These are shown in panel b, and the full answer is given in panel c.

$MC(x) = 3 + 0.08x$ , and so (by the usual calculations) the intermediate-run supply function of each firm is

$$s(p) = \begin{cases} 0, & \text{if } p < 3, \text{ and} \\ 12.5(p - 3), & \text{if } p \geq 3. \end{cases}$$

Therefore, total intermediate-run supply  $S_{\text{intermediate-run}}(p) = 12s(p)$ , which is

$$S_{\text{intermediate-run}}(p) = \begin{cases} 0, & \text{if } p < 3, \text{ and} \\ 150(p - 3), & \text{if } p \geq 3. \end{cases}$$

Intermediate-run supply equals demand (clearly, at a price above 3) if

$$150p - 450 = 2400 - 200p \quad \text{or} \quad 350p = 2850 \quad \text{or} \quad p_{IR} = 8.14.$$

At this price, total supply (which is the same as total demand) is 771.43; each of the 12 firms supplies 64.29, for total revenue 523.47, total cost 458.19, and (therefore) profit equal to 65.27.

In the long run, firms, attracted by the profits in this industry, begin to enter. They continue to do so until price falls to  $p_{LR} = 7$ , so that profit for each active firm is 0. Price is 7 when total demand is  $200(12 - 7) = 1000$ . This, then, is also total supply. Since each active firm supplies 50 units, this gives us 20 active firms in total, or eight entrants. Profit, of course, is 0 per active firm.

**14.8** I begin the solution by assuming that the fixed costs cannot be avoided by exiting the industry. Later I'll see if that assumption is relevant to my solution.

The four superior firms each have the marginal cost function  $MC(x) = 1 + 0.08x$ , so their (individual) supply functions are

$$s_{\text{superior}}(p) = \begin{cases} 0, & \text{if } p < 1, \text{ and} \\ 12.5(p - 1), & \text{if } p \geq 1. \end{cases}$$

Each of the eight other firms supplies according to the supply function computed last problem, or

$$s_{\text{other 8}}(p) = \begin{cases} 0, & \text{if } p < 3, \text{ and} \\ 12.5(p - 3), & \text{if } p \geq 3. \end{cases}$$

Overall supply is the sum of four of the first type and eight of the second, or

$$S(p) = \begin{cases} 0, & \text{if } p < 1, \\ 50(p - 1), & \text{if } 1 \leq p < 3, \text{ and} \\ 50(p - 1) + 100(p - 3) = 150p - 350, & \text{if } p \geq 3. \end{cases}$$

We have to find where this supply function intersects the demand function

$$D(p) = 200(10 - p).$$

At this point, it is usually a good idea to move to a graph of supply and demand to get a sense of where supply and demand intersect. But let me instead be a bit clumsy and just try trial and error.

Does the intersection occur on the segment of the supply function where  $S(p) = 50(p - 1)$ ? If it does, then  $50(p - 1) = 2000 - 200p$  or  $250p = 2050$ . This gives  $p = 8.2$ , which is outside the range of prices for which  $S(p) = 50(p - 1)$ .

So the intersection must take place at  $p > 3$ , where  $S(p) = 150p - 350$ . Equating supply and demand gives

$$150p - 350 = 2000 - 200p \quad \text{or} \quad 350p = 2350 \quad \text{or} \quad p = \$6.714.$$

At this price, total demand equals total supply, which is  $D(6.714) = 657.2$ , which is divided as follows: Each of the four superior firms produces  $12.5(6.714 - 1) = 71.425$ , and each of the eight other firms produces  $12.5(6.714 - 3) = 46.425$ . (If you check my math, you'll find that roundoff has produced a small discrepancy in total supply of 0.1 units.)

For each of the superior-technology firms, revenue is \$478.55, while cost is \$325.49, so that profit is \$154.06.

For each of the eight other firms, revenue is \$311.70, while cost is \$325.49, so that each of these firms sustains a loss of \$13.79.

In fact, we knew that these firms would sustain a loss as soon as we saw that the price was below 7, because in the previous problem, we saw minimum AC for firms with this technology is \$7.

So the assumption that firms cannot avoid their fixed costs is relevant. If the less-efficient firms could avoid their fixed costs by exiting, they would choose to do so. As they exit, the price is pushed up, until it reaches \$7. At that price, each of the four firms with the better technology produces  $12.5(7 - 1) = 12.5 \times 6 = 75$  units, for a total supply from them of 300. This provides each with \$175 in profit. Demand at \$7 is for 600 units, so we need 300 units supplied by firms with the second technology. We know from the previous problem that their efficient scale is 50, so we need six of them producing. That is the equilibrium if firms can avoid their fixed cost by shutting down.

**14.9** There is free entry and exit of the less efficient firms, so the answer is just what is in the final paragraph of the solution of Problem 14.8: The price is \$7, the four firms with the superior technology produce 75 units each for a total supply from them of 300, giving each a profit of \$175. Six of the less efficient firms are active, each producing at their efficient scale of 50, giving

another 300 units of supply to just meet the demand (at a \$7 price) of 600 units in total.

**14.10** Average revenue for a competitive firm is just the (constant) price  $p$  that prevails in the market. So profit margin is biggest when average cost is at its minimum; i.e., at efficient scale. The first statement is, therefore, a special case of the second statement.

**14.11** I begin with the spread sheet I used to get the numbers given in the text for a sunk cost of entry of \$15,000. Follow along on Figure S14.1. In Column A I enter a value for  $N$ , the number of active firms. Column B contains the corresponding level of production by a single firm, using the solution for  $x$  given in the display on page 336 of the text,  $x = 95,000/(20 + N)$ . Then, using the obvious formulae, I compute the individual firm's average cost (column C), industry-wide production (column D), market price (column E), individual firm monthly revenue (column F), total cost (column G), and monthly cash flow or profit (column H).

	A	B	C	D	E	F	G	H
	$N$	quantity $x$	AC at $x$	$N$ times $x$	price	firm monthly revenue	firm month total cost	firm monthly cash flow (or profit)
1								
3	100	791.66667	13.042982	79,166.67	\$20.83	\$16,493.06	\$10,325.69	\$6,167.36
4	218	399.15966	9.242123	87,016.81	\$12.98	\$5,182.37	\$3,689.08	\$1,493.28
5	219	397.48954	9.2264743	87,050.21	\$12.95	\$5,147.41	\$3,667.43	\$1,479.98
6	266	332.16783	8.622731	88,356.64	\$11.64	\$3,867.55	\$2,864.19	\$1,003.35
7	267	331.01045	8.6122098	88,379.79	\$11.62	\$3,846.41	\$2,850.73	\$995.68
8	369	244.21594	7.8516331	90,115.68	\$9.88	\$2,413.91	\$1,917.49	\$496.41
9	370	243.58974	7.8464238	90,128.21	\$9.87	\$2,404.67	\$1,911.31	\$493.36

Figure S14.1. The spreadsheet for the sunk-cost-of-entry model.

In row 3, I try an initial number of firms,  $N = 100$ . Firm month cash flow is over \$6000, which will attract entry even if the sunk cost of entry is \$150,000. Rows 4 and 5 show the values of  $N$  that bracket a monthly cash flow of  $\$150,000/101 = \$1485.15$ , rows 6 and 7 show the values that bracket a monthly cash flow of  $\$100,000/101 = \$990.99$ , and rows 8 and 9 show the values of  $N$  that bracket a monthly cash flow of  $\$50,000/101 = \$495.05$ .

Note that as the sunk cost of entry decreases, (a) the equilibrium number of firms increases (toward 930, the equilibrium number if there were no sunk cost of entry), (b) the equilibrium price declines (toward \$7, the long-run equilibrium price if there were no sunk cost of entry), (c) individual firm output declines (toward the monthly efficient scale of 100, which is what firms would produce if there were no sunk cost of entry), and (d) firm monthly cash flow declines (toward \$0, which is what would be the equilibrium value with no sunk cost of entry.)

## Industry-wide input-price effects

Even if all firms in a competitive industry have access to the same U-shaped average-cost technology and entry and exit are free, long-run supply may not be flat. The reason (perhaps I should say, one possible reason) is that while each individual firm may believe that it has access to as much of an input to production as it desires at the market price, as the industry *as a whole* increases its output, it increases demand for that input, driving up the input's price. This raises the minimum average cost of each firm in the industry. Here is a toy example that illustrates this possibility. (While this is a toy example, it is quite relatively difficult. Proceed with caution.)

Imagine a competitive industry in which all the firms have a production technology given by the production function  $f(k, l) = 20k^{1/4}l^{1/4}$ , for two inputs,  $k$  and  $l$ . To begin our analysis, imagine that there are 40 firms in this industry and that neither entry nor exit is possible. We'll assume (for now) that the firms have no fixed costs.

The price of  $k$  is a constant \$100 per unit. The price of  $l$  depends on how much  $l$  is demanded by the industry as a whole; at the current equilibrium, this price is \$100.

Each firm believes that it has very little impact on either the price  $p$  it gets for its output—this is the usual assumption of perfect competition—or the price  $r_l$  of the input  $l$ . So each firm chooses its production quantity and plan in a fashion that treats these prices as constant.

In the manner of Chapter 11, we conclude: If  $r_l = 100 = r_k$ , to make  $x$  units of output an individual firm will set  $k = l = x^2/400$ ; therefore, for each firm,  $TC(x) = 200x^2/400 = x^2/2$ , and  $MC(x) = x$ . Each firm sets  $p = MC$ , which gives individual firm supply functions  $s(p) = p$ . Industry supply is

$S(p) = 40s(p) = 40p$ . If we suppose that demand is given by  $D(p) = 20(60-p)$ , then supply equals demand is  $20(60-p) = 40p$ , or  $p = 20$ . Each firm produces 20 units of the product, taking 1 unit of  $k$  and 1 unit of  $l$ . Each firm's total cost is \$200 and its total revenue is \$400, so its profit is \$200.

Now the question arises, What would be the new equilibrium if demand shifts out to, say,  $20(75-p)$ ? Is it where supply intersects demand; that is, where  $20(75-p) = 40p$ ? For reference sake, the solution to this equation is  $p = 25$ .

The reason the answer is not a simple Yes, and the reason for this entire discussion, comes now. We supposed that the price of  $l$  is \$100. But suppose that this price holds when industry demand for  $l$  is 40 units. To be more specific, suppose that the inverse supply function of  $l$  to this industry is  $\hat{r}_l(l) = 60 + l$ . The concept of an inverse supply function is new, but I hope it is not mysterious: This says what price for this input is needed, as a function of industrywide supply of this input, to call out that level of supply from the industry that makes  $l$ .

Suppose the price of  $l$  is  $r_l$ . Suppose that each of the 40 firms believe that it can purchase as much  $l$  as it wishes at this price. (This is part of the assumption that the firms are competitive.) Now the solution to the firm's cost-minimization problem (the stuff of Chapter 11) is  $k = r_l l / 100$ , so  $x = 20(r_l / 100)^{1/4} l^{1/2}$ . Hence,  $l = 10x^2 / (400r_l^{1/2}) = x^2 / (40r_l^{1/2})$  and  $k = r_l^{1/2} x^2 / 4000$ . Therefore  $TC(x) = r_l^{1/2} x^2 / 20$ , and  $MC(x) = r_l^{1/2} x / 10$ . Supply by a single firm is therefore  $s(p) = 10p / r_l^{1/2}$ , and industry supply is  $S(p) = 400p / r_l^{1/2}$ .

Industry demand has shifted to  $20(75-p)$ , so supply will equal demand where  $400/r_l^{1/2} = 20(75-p)$ , or  $r_l = 20/(75-p)$ . Individual supply is  $x = 10/r_l^{1/2}$ , and since individual demand for  $l$  is  $x^2/(40r_l^{1/2})$ , individual-firm demand for  $l$  is  $2.5/r_l^{3/2}$ . Hence  $r_l = 60 + 40(2.5/r_l^{3/2}) = 60 + 100/r_l^{3/2}$ . This, unhappily, is a quintic equation, so I resorted to Excel to find a solution: It is at  $p = 26.2075$  (not  $p = 25!$ ), approximately, at which point  $r_l$  has risen to 115.402 and individual supply is 20.385, for an industry supply of 975.85. Individual demand for  $l$  is 1.385, so industry demand for  $l$  is 55.402.

Go back to the original situation, where  $p = 20$  and  $r_l = 100$ . Each firm's marginal-cost function is  $MC(x) = x$ , and so the horizontal sum of the 40 individual-firm supply functions is  $S(p) = 40p$ . But this *does not* take into

account the industry-wide impact of increased supply on  $r_l$ . What is the true supply function? We can plot it numerically as follows. Suppose the market price of the output good is  $p$ , and the price of  $l$  is  $r_l$ . The individual firm's output is  $x = 10p/r_l^{1/2}$ , and its demand for  $l$  is

$$\frac{x^2}{40r_l^{1/2}} = \frac{100p^2/r_l}{40r_l^{1/2}} = 2.5 \frac{p^2}{r_l^{3/2}}.$$

Hence total industry demand (as a function of  $p$  and  $r_l$ ) is  $100p^2/r_l^{3/2}$ . This gives us an equation for  $r_l$ :

$$r_l = 60 + \frac{100p^2}{r_l^{3/2}} \quad \text{or} \quad \frac{r_l^{5/2} - 60r_l^{3/2}}{100} = p^2 \quad \text{or} \quad p = \frac{\sqrt{r_l^{5/2} - 50r_l^{3/2}}}{10}.$$

Hence, using Excel, we can parametrically vary  $r_l$  and compute the corresponding (equilibrium) values for  $p$  and  $x$ ; we then plot  $x$  against  $p$  to get the "true supply function." See Figure S14.2. Both the true supply function and the supply function one obtains by horizontal adding the individual firm's supply functions (for  $r_l = 100$ ) are shown. Note that the true supply function is "steeper"; this reflects the fact that, as industry output rises,  $r_l$  rises and so, for output prices above 20, the firms supply less than they would with  $r_l = 100$ ; for output prices below 20,  $r_l$  is less than 100 and so firms supply more.

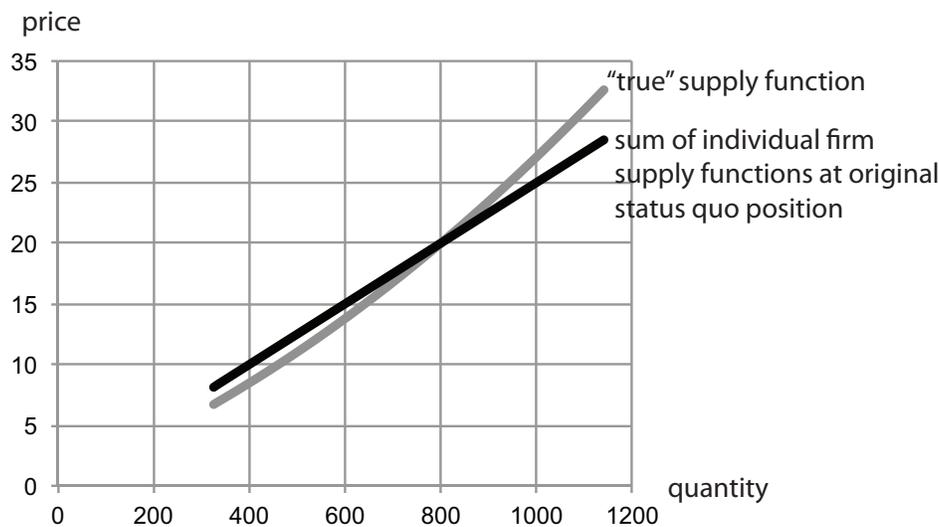


Figure S14.2. "True" versus notional supply. The gray function is the "true" industry supply curve, taking into account the impact total industry supply has on the price of input good  $l$ .

Now change the model as follows: Instead of a fixed number of firms, assume there is an unlimited number of firms, all having access to the same technology, with (in the long run) conditions of free entry and exit. (For simplicity, we will not include a sunk cost of entry.) Since marginal cost is rising, to keep from reaching an “equilibrium” in which infinitely many firms are producing infinitesimal amounts, we assume that firms that are active must pay a fixed cost of \$200. (The value \$200 is chosen for reasons that will become clear in a bit.)

Now, in any long-run equilibrium (the only sort with which we’ll be concerned here), price must equal minimum average cost. As a function of  $r_l$ , total cost is given by

$$200 + \frac{r_l^{1/2}}{20}x^2,$$

and so efficient scale (where average cost equals marginal cost) is where

$$\frac{200}{x} + \frac{r_l^{1/2}}{20}x = \frac{r_l^{1/2}}{10}x \quad \text{or} \quad x = \frac{20\sqrt{10}}{r_l^{1/4}}.$$

The corresponding minimum average cost is

$$p = \frac{r_l^{1/2}}{10} \times \frac{20\sqrt{10}}{r_l^{1/4}} = 2\sqrt{10}r_l^{1/4}.$$

Given  $r_l$ , a firm producing  $x$  efficiently employs

$$l = \frac{x^2}{40r_l^{1/2}},$$

so firms producing at efficient scale (which is  $x = 20\sqrt{10}/r_l^{1/4}$ ) employ

$$l = \frac{100}{r_l} \text{ units of } l.$$

Supposing total industry output is  $X$ , we need

$$\frac{X}{20\sqrt{10}/r_l^{1/4}} = \frac{Xr_l^{1/4}}{20\sqrt{10}} \text{ firms,}$$

and so total demand for  $l$  is

$$\frac{5X}{\sqrt{10}r_l^{3/4}},$$

and so

$$r_l = 60 + \frac{5X}{\sqrt{10}r_l^{3/4}} \quad \text{or} \quad X = \frac{\sqrt{10}}{5} \left[ r_l^{7/4} - 60r_l^{3/4} \right].$$

As before, we can parametrically vary  $r_l$  and get corresponding (equilibrium) levels of  $p$  and  $X$ , and use those to plot the long-run (free-entry) supply function.

See Figure S14.3. The gray curve is the “true” free-entry industry supply curve. As industry output rises, so does the price of  $l$ , because demand for  $l$  rises, and so minimum average cost (which is the equilibrium price) rises.

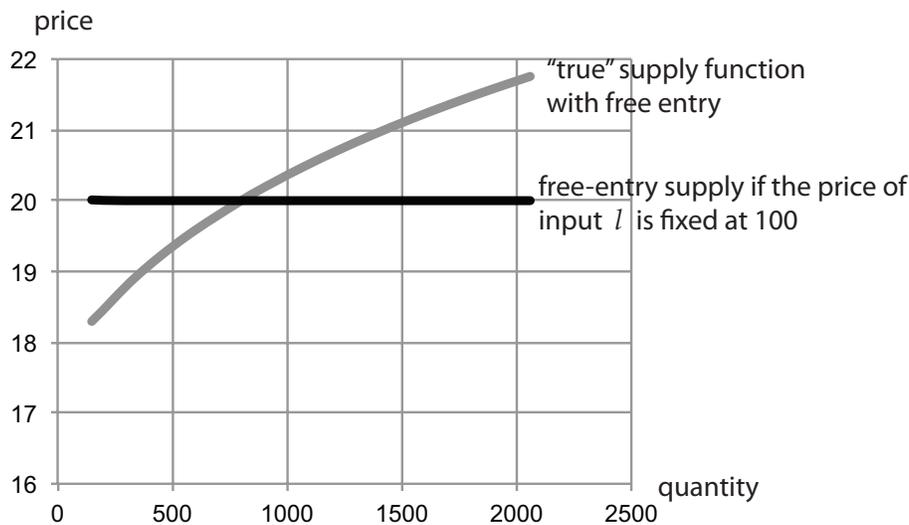


Figure S14.3. “True” supply with free entry.

Note that at  $r_l = 100$ , the formulae

$$x = \frac{20\sqrt{10}}{r_l^{1/4}}, \quad p = 2\sqrt{10}r_l^{1/4}, \quad \text{and} \quad X = \frac{\sqrt{10}}{5} \left[ r_l^{7/4} - 60r_l^{3/4} \right]$$

give  $x = 20$ ,  $p = 20$  and  $X = 800$ . (And, therefore, the number of firms in the free-entry equilibrium is 40.) So the conditions initially described in this exercise—40 active firms, each producing 20 units, with equilibrium output price 20 and equilibrium price of  $l$  equal to 100—is the free-entry equilibrium as well with the fixed cost of 200. (That's why I chose 200.) So, if you wanted to do an intermediate-run versus long-run analysis of how the industry responds to shifting demand, you would superimpose the two figures.

Two comments about this exercise to wrap up:

- Chapter 14 concerns what economists call *partial equilibrium analysis*, studying one industry while assuming all other prices are fixed. This discussion and exercise have begun to discuss *general equilibrium effects*, where what happens in one market affects other markets and, in particular, affects prices in other markets. This is hard stuff. But, to the extent that real-life markets have these effects, to make accurate predictions about, for instance, how equilibrium price in one market would react to a shift in demand, you must consider several markets (in this case, two) simultaneously. (You'll get to do a few exercises of this sort in the next set of review problems, but we spend no time on them in the text.)
- Looking at either Figure S14.2 and Figure S14.3, there are two “supply functions,” one based on a simple-minded hypothesis that the input price does not change and one based on recognition that, as the scale of the industry output changes, so does this input price. I've used the adjective “true” for the supply function that takes into account the impact industry scale: If you want to make predictions about how prices would change if there were a shift in demand, then you want to take into account the impact of changes in  $r_l$ , so the two gray supply functions are what you employ. In the next chapter, however, we discuss something called *producer surplus*; in that case, the simple-minded supply function is the right one to use. If you want to know why, consult a doctoral-level book on the subject: Be on the lookout for what is known as *Hotelling's lemma*.<sup>1</sup>

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<sup>1</sup> For instance, see Kreps, *Microeconomic Foundations I: Choice and Competitive Markets*, Princeton University Press, 2013. See, in particular, the discussion on page 295 in the subsection labelled *Warning*.

## Monopolistic Competition

A monopolistically competitive industry or market comprises many suppliers (producers) and many demanders, but unlike perfect competition, the good in question is not a commodity but differentiated. That is, different consumers are more or less interested in the variety produced by a particular producer. For instance, Castro Street in Mountain View, California, is lined with restaurants of various types, some serving Indian food, some serving Chinese, some Italian, and so forth. And the Indian-cuisine restaurants are differentiated: Some serve food from northern India, while others specialize in the cuisine of southern India. A given consumer might have a preference for northern Indian food over everything else, and then rank Szechuan Chinese food second, followed by Sicilian Italian cuisine, then Mandarin Chinese, and so forth. A different consumer might prefer Lebanese cuisine most of all, followed by southern Indian, and so on.

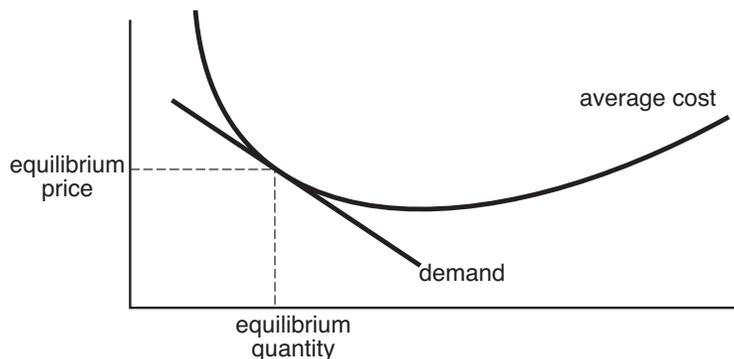
In this environment, there is no reason to believe that a single market price must prevail for “lunch.” If the northern Indian restaurant raises its price above the prices charged by the other restaurants, it loses some business. But consumers with a strong preference for northern Indian cuisine will pay the higher price. Despite the many suppliers, each restaurant has some market power, each faces a downward sloping demand function, and each maximizes profit by equating marginal cost and marginal revenue.

Of course, demand at any one restaurant is affected by the prices charged by all the others. If a northern Indian restaurant, call it the Tandoor, charges \$10 for lunch, and all the other restaurants charge \$5, the Tandoor is going to sell fewer lunches than if all the others charge \$12. The question is, How do the prices other restaurants charge affect the demand faced by the Tandoor?

In monopolistic competition, the demand facing any one producer, such as the Tandoor, depends on the full distribution of prices the others charge. If, say, Vesuvio, the Italian restaurant adjacent to Tandoor, lowers its prices, this has almost no effect on the demand at Tandoor. But, if all the other restaurants lower their prices, demand at the Tandoor falls. And, if new restaurants enter the market, this causes demand at the Tandoor to fall. At the same time, no matter what prices other restaurants charge or how many other restaurants there are, the Tandoor retains some market power; its demand function does not flatten out.

What does an equilibrium in this sort of market look like? Each firm is

assumed to maximize its own profit, so each sets its price and quantity to equate its marginal cost and marginal revenue. Moreover, it is usually assumed that there is free entry into the industry, entry drawn by the lure of positive economic profit. And firms exit if they can (at best) make a loss. Suppose, then, for simplicity, that all firms have U-shaped average cost curves. In a long-run equilibrium, with free entry and exit, the marginal (producing) firm must make 0 profit. *This means that the demand function facing this marginal firm must intersect the firm's average cost curve—otherwise, the firm would be unable to make nonnegative profit and would exit—but the demand function cannot cross through the firm's average cost curve—otherwise, the firm could make positive profit and other firms like it would enter.* In other words, the (marginal) firm's demand function must be tangent to the firm's average cost function, as in Figure S14.4. Since, by assumption, the firm faces downward sloping demand, it must be producing at less than its efficient scale.



*Figure S14.4. The marginal firm in a monopolistically competitive industry. The firm on the margin between entering and exiting must make 0 profit. This implies that the demand function it faces must just touch its average cost function, and since it faces downward sloping demand, it must be producing at less than its efficient scale.*

That is the basic idea. There is competition and free entry, but firms retain market power. This, the theory tells us, leads firms to produce below their efficient scale. (This is true for the marginal firm on the cusp between entry and exit. If we play semantic games with the concept of rent, it is true more generally for all firms in the industry.)

I go no further with monopolistic competition, because I find it hard to give real-life examples of industries that conform to its assumptions. Specifically,

the notions that demand facing a firm remains less than perfectly elastic, but no other firm's prices affect (by much) the first firm's demand, seem unlikely. A firm retains market power if its product is differentiated from the products of competitors; the Tandoor indeed has its own clientele. But the basis of that differentiation—location, perhaps, or cuisine style—means that “close competitors” have a sizeable impact on the demand facing the Tandoor. If the restaurant next door to the Tandoor or the northern Indian restaurant a block away changes its price, that has an impact on demand at the Tandoor. And once we suppose that this is true, we have to start thinking in terms of ideas from Part I; e.g., what is the nature of the relationship between the owner of the Tandoor and the owner of the India Palace, another Northern Indian restaurant in the neighborhood.

Why bother you with this theory at all? There are two reasons. First, the rise of internet-based marketing might, in the case of some products, lend increased credibility to this model. And, the models of monopolistically competitive markets possess features that make them very useful in some economic contexts, most notably in macroeconomic theories of trade and economic growth. You may run into these models in a course in macroeconomics, and to avoid angering your macroeconomics professor, I provide this brief introduction. But, until you encounter monopolistic competition in some other course, my suggestion is to forget everything covered in the preceding discussion. You certainly do not need these ideas for the remainder of the text.