

Chapter 15 Material

Solutions to the problems of Chapter 15 are presented. Following the solution to Problem 15.7, a general discussion of P.I.E. and the Four Slices (and how it compares to Porter's Five Forces) is provided. And following this is a discussion of alternative and more general proofs of the (allocative) efficiency of competitive markets.

15.1 The first step is to draw the picture of supply equals demand. See Figure S15.1. The market equilibrium is where

$$S(p) = 1000(p - 4) = 3000(20 - p) = D(p),$$

or $p - 4 = 3(20 - p)$, or $p - 4 = 60 - 3p$, or $4p = 64$, or $p = 16$. This corresponds to a quantity of $1000(16 - 4) = 12,000$. Therefore, consumer surplus is the area of a right triangle whose height is $20 - 16 = 4$ and whose base is 12,000, which is \$24,000. And producer surplus is the area of a right triangle whose height is $16 - 4 = 12$ and whose base is 12,000, or \$72,000.

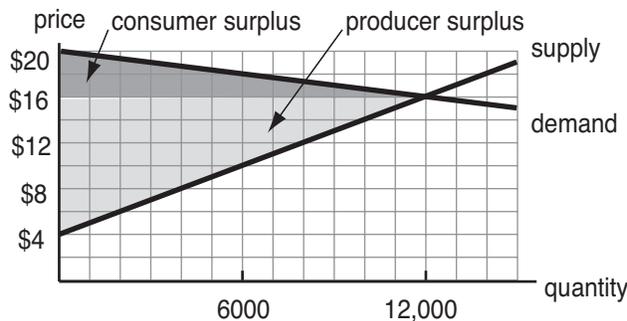


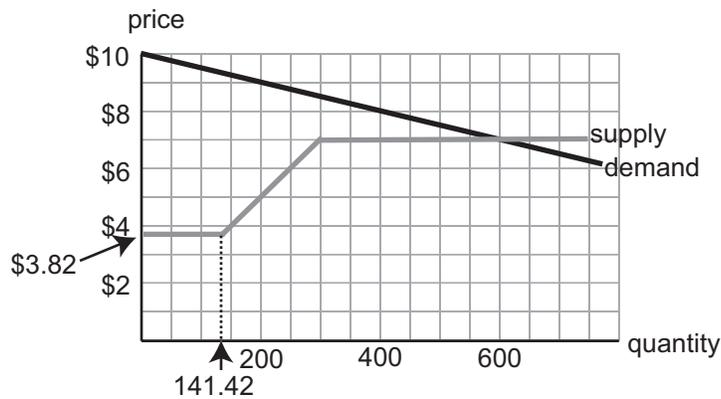
Figure S15.1. Solving Problem 15.1. Supply and demand are plotted and consumer and producer surpluses are the areas of the triangles indicated.

15.2 The first step is to draw the supply-equals-demand picture for this economy. This is shown in Figure S15.2(a). Now we are assuming that all fixed costs are avoidable, so the four more efficient firms begin to supply

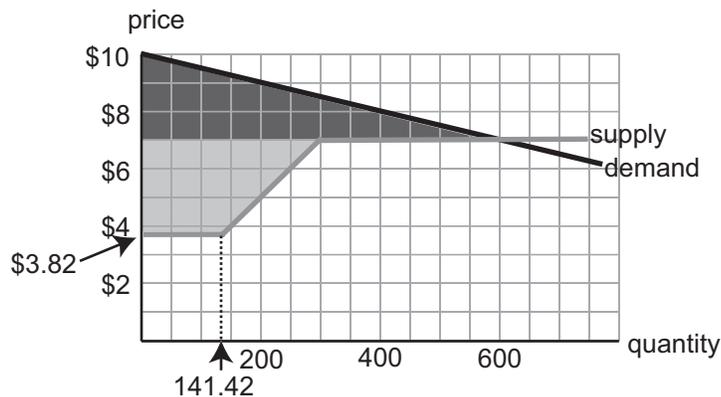
when price reaches their minimum average cost. We get this in the usual fashion: Find efficient scale by setting

$$\frac{50}{x} + 1 + 0.04x = 1 + 0.08x \quad \text{or} \quad x^2 = \frac{50}{0.04},$$

which gives $x = 35.355339$. Minimum average cost is then $1 + 0.08 \times 35.355339 = \3.8284 . Each firm supplies its efficient scale at that price, for a total supply of 141.42. Thereafter, these four firms supply along their marginal cost curves. When the price reaches \$7, there is (potentially) infinite supply from the long line of less efficient firms. Demand is obvious, and we get the equilibrium: 600 units in total; 300 coming from the 4 more efficient firms, or 75 units apiece, for a net profit of \$175 each, or \$700 total; 300 units from six of the less efficient firms, each of whom produces at efficient scale (for them) of 50, each making 0 profit.



(a) Supply equals demand



(b) Consumer and producer surplus

Figure S15.2. Problem 15.2.

In Figure S15.2(b) we shade in consumer and producer surplus, in darker and lighter shades, respectively. Consumer surplus is a right triangle with a height of \$3 and a base of 600 units, for a total consumer surplus of \$900. Producer surplus is a quadrilateral with height \$3.1716 and two parallel bases of 141.42 units and 300 units, for a total area of

$$3.1716 \times \left(\frac{141.42 + 300}{2} \right) = 3.1716 \times 220.71 = \$700.$$

Producer surplus is precisely the sum of the profits of the firms, because in this instance supply is drawn assuming fixed costs can be avoided.

Suppose we had assumed that the four more efficient firms could not avoid their fixed costs. Then their supply would begin at their minimum marginal cost, which is \$1, and supply would look like the picture, except that supply would extend down to 0 supplied at $p = \$1$. Then producer surplus would be the area of a right triangle with a height of \$6 and a base of 300 units, which is \$900. This is the profits of the four firms, but now gross of their fixed costs of \$50 apiece, since now the fixed costs are assumed to be unavoidable.

15.3 Figure S15.3 recaps the solution from Problem 14.7: Long-run supply is flat, with as much quantity as needed at a price of \$7. Hence, the initial equilibrium is where price is \$7 and quantity is 600. Intermediate-run supply is shown as a medium gray line, and short-run supply, the light gray line, is vertical, fixed at 600 units. When demand shifts out,

- In the short run, the price rises to \$9 and the quantity stays fixed at 600 units.
- In the intermediate run, the price falls to \$8.14 and the quantity rises to 771.43 units.
- In the long run, the price falls back to \$7 and the quantity rises to 1000 units.

Consumer surpluses are computed next:

- At the old equilibrium, the consumer surplus is the area of a right triangle with height \$3, from \$7 to \$10, and base 600 units, or \$900.
- When demand shifts, the consumer surplus in the short run is the area of a right triangle with height \$3, from \$9 to \$12, and a base of 600 units, or \$900.
- In the intermediate run, the consumer surplus becomes the area of a right triangle with height \$3.86, from \$8.14 to \$12, and a base of 771.43 units,

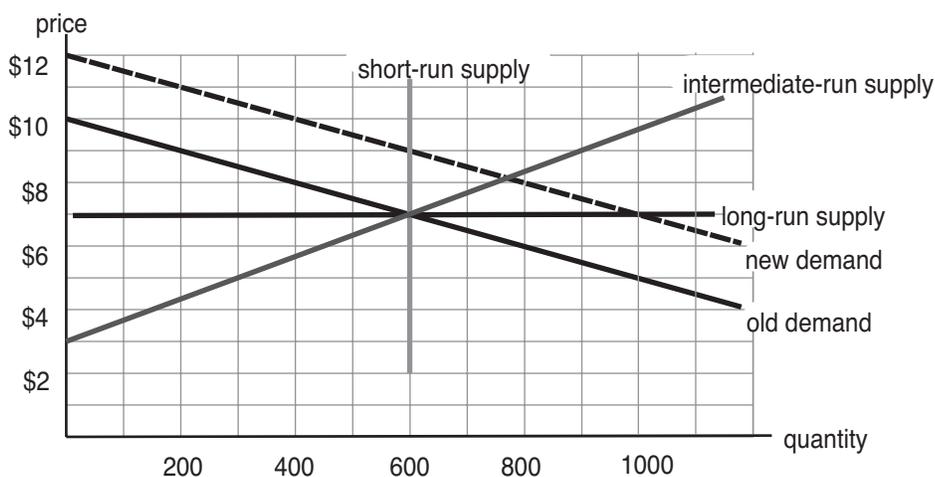


Figure S15.3. Problem 15.3: *The big picture*. This figure graphs the three supply curves (from the status-quo, old equilibrium), the old demand function, and the new demand function.

or \$1488.86.

- Finally, in the long run, consumer surplus becomes the area of a right triangle with height \$5, from \$7 to \$12, and a base of 1000 units, or \$2500.

Some care is needed in the computation of producer surplus. Let me begin by computing the three producer-surplus figures for the original equilibrium.

- The status-quo equilibrium price and quantity are 600 units and \$7. The short-run supply function is vertical (supply = 600 regardless of price), so in the short-run, status-quo producer surplus is the area of the rectangle in panel b of Figure S15.4, which is $\$7 \times 600 = \4200 .
- The intermediate-run supply function is the function $S(p) = 150(p - 3)$. So intermediate-run producer surplus is the area of the shaded triangle in panel d of Figure C15.4, a right triangle with a base of 600 units and a height of \$4, for an area of $(1/2) \times 600 \times \$4 = \$1200$.
- And the long-run supply function is flat at \$7: There is no supply in the long-run at any price less than \$7, and as much supply as the market demands at $p = \$7$. So the “area” that is producer surplus is the area of a line-segment of zero height and length 600; the area is \$0, as is the area of all line segments.

This seems to be saying that short-run producer surplus at the status quo is larger than intermediate-run producer surplus, which is larger than long-run producer surplus. *But remember*, producer surplus in any -run is the sum

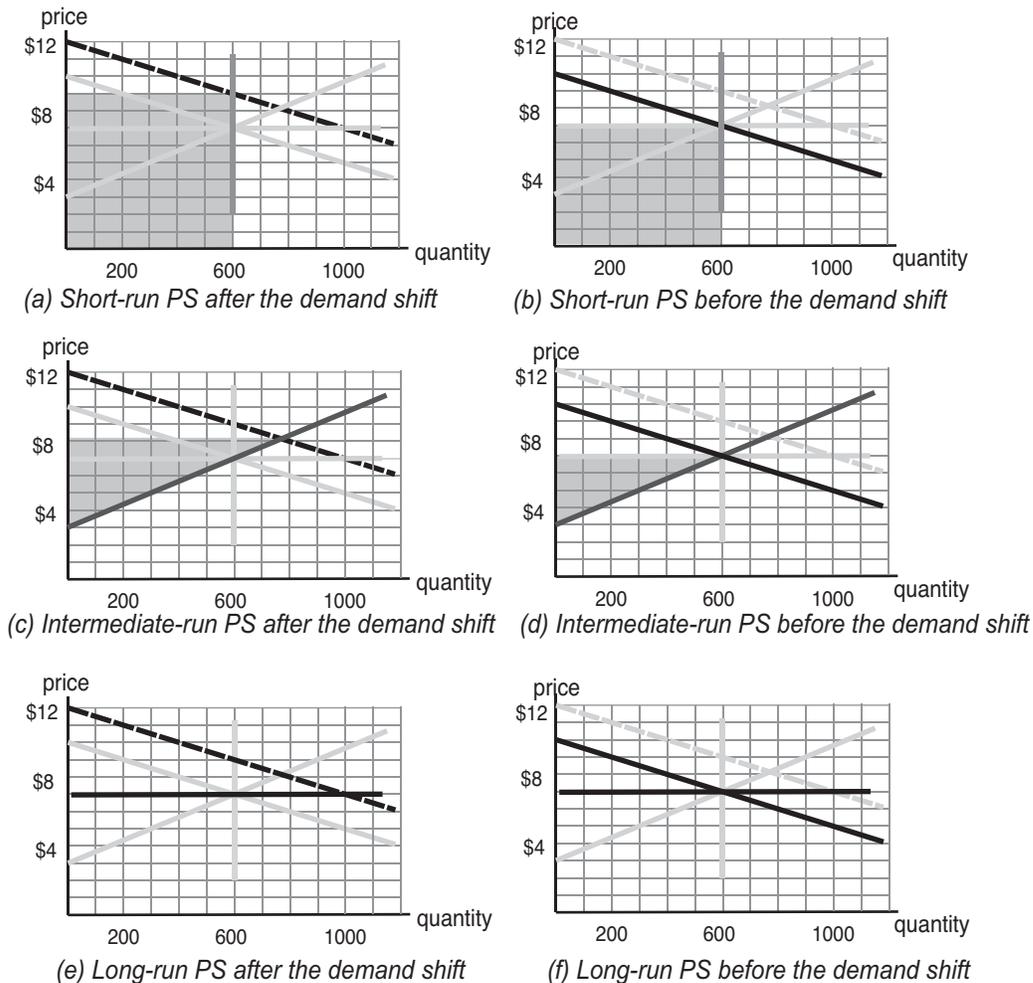


Figure S15.4. Problem 15.3. Short-run, intermediate-run, and long-run producer surpluses before and after a shift in demand. To find each run's producer surplus (PS) for a particular run, find the areas bounded by that run's supply curve and the equilibrium price, out to the equilibrium quantity. These are profits gross of any fixed costs for the particular run. Panels a and b show the short-run producer surpluses, after and before the demand shift; panels c and d show the pictures for the intermediate run. Panels e and f are for the long run; since long-run supply is horizontal, producer surplus is always zero.

of the firm's profits *plus* any and all fixed costs that are unavoidable in that -run.

- In the long-run, no fixed costs are unavoidable and, in this market with free entry at the best available technology, it is of course true that \$0 is the sum of all the firms' profits. This is panel f in Figure S15.4.
- In the intermediate run, firms can neither enter nor depart. Since there are 12 firms and each firm has a fixed cost of 100, *which they cannot avoid*,

unavoidable fixed costs are \$1200. Intermediate-run producer surplus is indeed the profit made by the 12 firms at the status quo (namely \$0), with unavoidable fixed costs of \$1200 added back in.

- And in the short-run, the 12 firms can't do anything at all except produce at their status-quo levels. *All their costs are fixed.* That's the \$4200 figure that we are calling short-run producer surplus: Note that each firm makes zero profit (net of its costs), and total revenues for the industry are 7×600 units, or \$4200. Just as it should be.

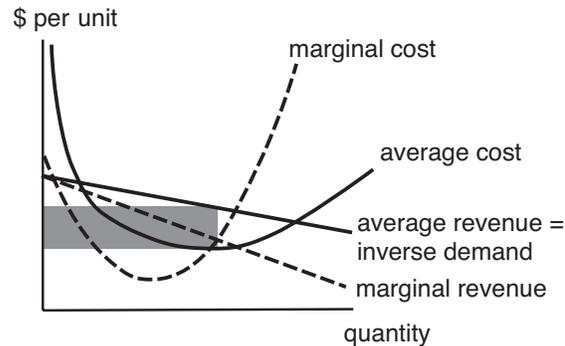
Now we shift demand out. Panel a gives the short-run picture and, in particular, the shaded area is short-run producer surplus. Panel c shows intermediate-run producer surplus. And panel e shows (well, it sort of shows) long-run producer surplus.

- In panel a, the shaded area is a rectangle with a base of 600 units and a price of \$9, for an area of \$5400. This is the profits of the firms gross of their fixed costs. *And we find the net increase in their profits by subtracting the shaded area in panel b from the shaded area in panel a.* Since both of these are profits gross of short-run fixed costs, when we subtract, the short-run fixed costs cancel out. The net change in short-run profits is $\$5400 - \$1200 = \$4200$.
- In panel c, the shaded area is a right triangle with with height \$5.14, from \$8.14 down to \$3, and base 771.43 units, or \$1982.57. This is profits gross of intermediate-run fixed costs; and if we subtract off the triangle from panel d, we cancel those out: the net change in profits in the intermediate run is $\$1982.57 - \$1200 = \$782.57$.
- Finally, in the long run, producer surplus after the shift in demand is \$0 (panel e, and there is no shaded area to look for here because the appropriate "area" has zero height), whereas before the shift, long-run producer surplus was \$0. So, in the long run, producer profit changes by \$0.

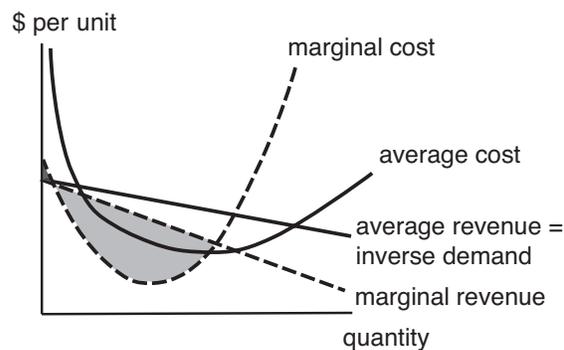
Two comments, here: (1) Go back to page 348, and start re-reading from start of that page. You don't understand what this problem is telling you until you reach an "Aha!" moment concerning this problem, as in "Aha! Producer surplus in any -run is profit plus any fixed costs that are avoidable in that -run. But if you subtract X-run producer surplus before some change in conditions from X-run producer surplus after the change, the difference is the change in profits, since the subtraction cancels out those unavoidable fixed costs."

And (2) In this sort of industry, when demand shifts out, firms do best (see their profits rise the most) the less they can do to react to the shift. In the short run, they can't change supply, so price shoots up, and it is all windfall profit for them. In the intermediate run, the firms lucky enough to be in the industry start to produce more, driving down the price and, because they are competing, driving down their windfall profits. And in the long run, as more firms enter, everyone's profits are driven down to zero, at least when there is free entry at the best available technology.

15.4 See Figure S15.5. Note that another way to "see" gross profit is as the rectangle formed from average revenue that gives total revenue (the rectangle formed by the origin and the profit-maximizing price and quantity) less the area under the marginal cost function.



(a) *Net profit*: The shaded region is a rectangle with height equal to average revenue less average cost and with base equal to the profit-maximizing quantity. This is the net profit.



(b) *Gross profit*: One way to "see" gross profit is as the area between marginal revenue and marginal cost. Note that in this case, this is the lightly shaded area, where MR exceeds MC, less the small darkly shaded triangle, where MC exceeds MR.

Figure S15.5. Problem 15.4.

15.5 Inverse demand is given by $P(x) = 20 - x/3000$, so marginal revenue is given by $MR(x) = 20 - x/1500$. Marginal revenue equals marginal cost

where

$$20 - \frac{x}{1500} = 4 + \frac{x}{1000} \quad \text{or} \quad 16 = \frac{5x}{3000} \quad \text{or} \quad x = 9600,$$

for a price of \$16.80. (See Figure S15.6.)

- The consumer surplus is the area of a right triangle with height \$3.20, from \$16.80 to \$20, and base 9600 units, or \$15,360. (Compare this with consumer surplus of \$24,000 in Problem 15.1.)
- The producer surplus is the area of a right quadrilateral, with base 9600 units, and parallel sides \$12.80 (from \$16.80 to \$4) and \$3.20 (from \$16.80 to \$13.60), or $9600(12.8 + 3.2)/2 = \$76,800$. (Compare this with producer surplus of \$72,000 from Problem 15.1.)

Although you weren't asked to do this computation, note that total surplus in Problem 15.1 was \$96,000, while here it is \$92,160, a difference of \$3840. This is the area of the deadweight loss triangle indicated in Figure 15.16: a base of \$3.20 and a height of 2400 units, for an area of \$3840.

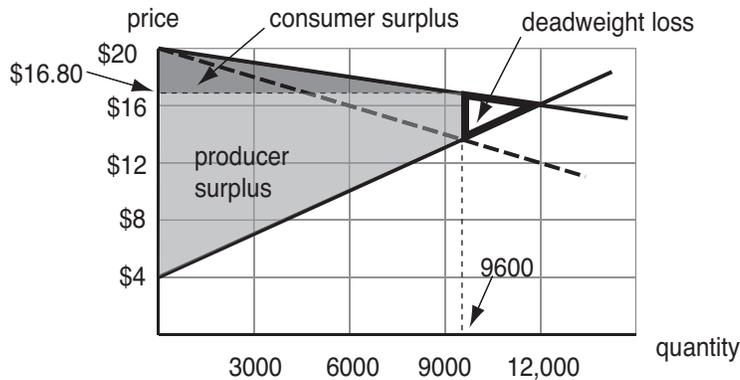


Figure S15.6. Solving Problem 15.5. Consumer surplus, producer surplus, and the deadweight loss are as marked.

15.6 (a) If there were 20 million pairs of shoes available, consumer surplus would be the area of a right triangle with height \$80 (or whatever is the currency) and base 20 million shoes, or \$800 million. But with this rationing scheme, and with every consumer lucky enough to get a pair of shoes wearing them, only half the consumers get to enjoy this surplus. If the allocation mechanism is random, the law of large numbers (or the law of averages) suggests that a representative sample of consumers will get shoes, representative in terms of how they value the shoes, so only \$400 million (half of the \$800 million) in consumer surplus will be enjoyed.

(b) The black market, if it functions well, will get the 10 million shoes onto the feet of the consumers who value the shoes most highly—the black market price will be \$50 a pair. *Money will change hands among consumers, but in terms of consumer surplus, that will be a net wash.* Total consumer surplus will be the gross value in dollars to consumers of those shoes—the area of a quadrilateral with a base of 10 million shoes and heights perpendicular to that base of \$90 and \$50, for an area of \$700 million—less the cost of those shoes (paid to the government producer) to consumers, which is \$10 per pair times 10 million pair, or \$100 million. Net consumer surplus is therefore \$600 million.

It may be worth thinking for a moment about how long the queues will be at the state shoe stores, if it is anticipated that there will be a well-functioning black after-market. Consumers—speculators might be a better term—can anticipate that the black-market price will exceed the \$10 price in the state shoe stores, even if they can't predict an equilibrium price of \$50. So even consumer-speculators who have no personal use for a pair of shoes will queue up at the state shoe stores, hoping to get a pair in anticipation of reselling them and making a bit of money. And, needless to say, the salespersons in the state shoe stores may set aside a few dozen pairs for their own use (as shoe speculators) or for the use of friends and family. So while black markets may increase overall consumer surplus, the “costs” in terms of accompanying corruption are significant. And insofar as such corruption enters negatively into the utility functions of the populace, a broader definition of “the consumer surplus generated by shoes” could well temper the notion that black markets are good for maximizing consumer surplus.

15.7 (a) If each firm has total cost function $TC(x) = 4x + x^2/200$, then the marginal cost function is $MC(x) = 4 + x/100$. Since the firms are perfectly competitive, the supply function for each is obtained by solving

$$p = 4 + x/100,$$

giving individual supply function

$$s(p) = 100(p - 4)$$

for prices above 4. There are 25 such firms, so total supply is

$$S(p) = 2500(p - 4),$$

and supply equals demand where

$$2500(p - 4) = 10,000(10 - p) \quad \text{or} \quad 2500p - 10,000 = 100,000 - 10,000p \quad \text{or}$$

$$12,500p = 110,000 \quad \text{or} \quad p = \$8.80.$$

Total supply at this point (which equals demand) is

$$10,000(10 - 8.8) = 12,000.$$

See Figure S15.7(a). Consumer surplus is a triangle with base 12000 and height \$1.20, or area = consumer surplus = $(1/2)(1.2)(12000) = \$7200$. Producer surplus is a triangle with base 12,000 and height \$4.80, or area = producer surplus = $(1/2)(4.8)(12,000) = \$28,800$. Note that each of the 25 firms makes a profit of $\$28,800/25 = \1152 .

(b) Follow along on Figure S15.7(b). This \$1 tax effectively raises the marginal cost of the firm by \$1 per unit, to $MC(y) = 5 + y/100$. Therefore, supply equals demand where

$$2500(p - 5) = 10,000(10 - p) \quad \text{or} \quad 12,500p = 112,500 \quad \text{or} \quad p = \$9.$$

This reduces the amount supplied to $10,000(10 - 9) = 10,000$ units. Consumer surplus falls to

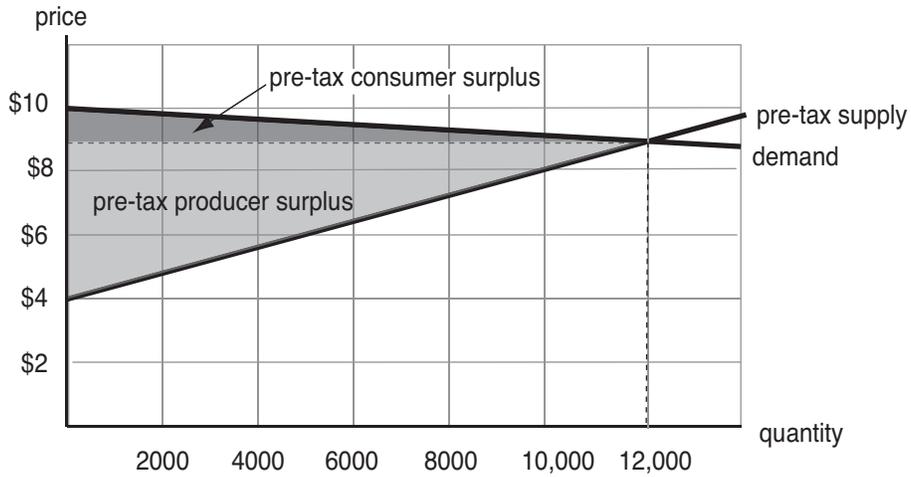
$$(1/2)(10,000)(1) = \$5000,$$

and producer surplus falls to

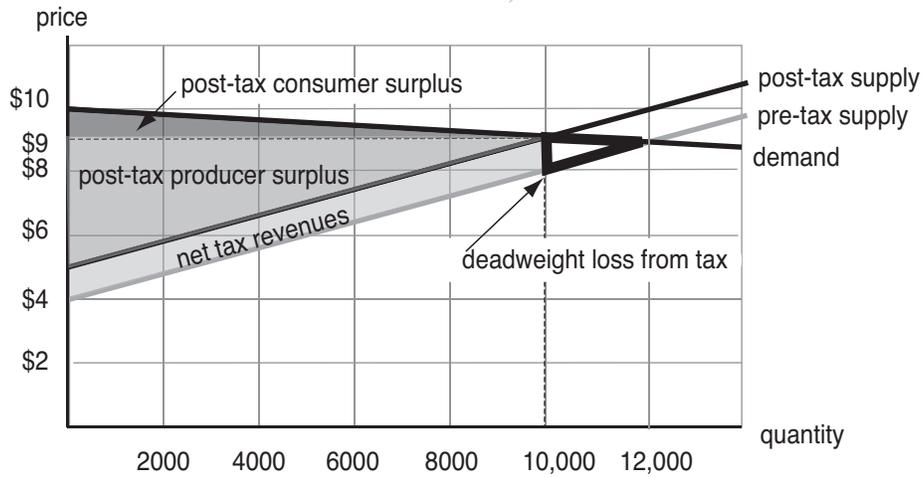
$$(1/2)(4)(10,000) = \$20,000,$$

or \$800 per firm. In addition, the government collects \$1 apiece on each of 10,000 units, for net revenues of \$10,000.

(c) Note that the total surplus in part a is $\$28,800 + \$7200 = \$36,000$, while in part b total surplus is $\$20,000 + \$5000 + \$10,000 = \$35,000$. The tax results in a \$1000 loss in total surplus; firms lose \$8800 of profit, consumers lose \$2200 worth of utility, and the government gets back only \$10,000. The lost \$1000 of surplus can be seen in Figure S15.7(b), where the post-tax consumer surplus, producer surplus, and government receipts from the tax are shaded in successively lighter shades. Comparing with the total surplus from before,



(a) The pre-tax situation: equilibrium, consumer surplus, and producer surplus



(b) Post-tax equilibrium, consumer surplus, producer surplus, tax revenues, and the deadweight loss in surplus from the tax

Figure S15.7. Equilibrium and surpluses before and after a tax.

what is missing is the little triangle that is heavily outlined. The surplus in this triangle is “lost” because the tax causes an inefficiently small amount of the good to be produced; firms stop short of the point where marginal cost gross of the tax equals marginal benefit.

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P.I.E. and the Four Slices

In Saloner, Shepard, and Podolny, *Strategic Management*, (Wiley, 2005), the

basic concepts that Michael Porter covers with his Five Forces are translated into P.I.E. and the Four Slices. Recall that the question Porter posed and to which he proposed *The Five Forces* as where to look for answers: *How profitable (generally) are firms in a given industry?* P.I.E. and the Four Slices is, essentially, a two-step refinement of Porter's answer.

P.I.E., or the Potential Industry Earnings, is the answer to the question *How much value (measured in dollar terms) is the industry able to produce for society, assuming it operates efficiently?* The picture of P.I.E. is Figure S15.8.

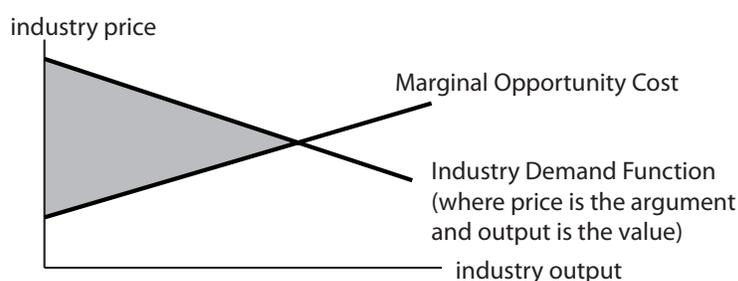


Figure S15.8. P.I.E.

The term *marginal opportunity cost* is new, but it means, essentially, how much it costs society to produce (on the margin) each unit of the good. And with this, you should have no problem (following Chapter 15) in understanding why the shaded area in this figure bears this interpretation, albeit subject to the caveats we mentioned in the chapter about what consumer surplus really means: The shaded area is, of course, the total surplus that the market can generate.

The Four Slices are then advanced as the answer to a second question: *Given that P.I.E. is the value for society created by this industry, at least potentially, how much of that value is captured (in the form of profits) by firms in the industry?* The four slices are supplier power, customer power, entry-barrier costs, and losses due to rivalry issues:

- Supplier power, roughly, tells you how much above the marginal opportunity cost to society suppliers to the industry can extract from the industry. This is, more or less, identical to the same concept in Porter.
- Customer power tells you how much of the consumer (or customer) surplus the customers of the industry can extract. Again, pretty similar to the same concept in Porter, but keep reading.
- Entry-barrier costs reflect what firms in the industry must give up to keep entrants at bay. The parallels to Porter's entry barriers are apparent.

- Rivalry losses reflect what firms lose because they can't collude and do what is best for their collective selves.

This leaves Porter's force *substitutes and complements* unaccounted for: But the existence of substitute and complementary goods is what determines the position of the Industry Demand Function. If, say, there were good substitutes available, that would lower the level of demand to the industry at any given price. And if there are cheap complementary goods, that raises industry demand at any given price. So Porter's substitutes and complements is one important factor that goes into the position of the industry demand function, hence the size of the P.I.E.

That said, some comments are in order: P.I.E. is calculated based on the "notion" that firms within the industry can extract all the consumer surplus they create. It takes first-degree price discrimination to do that and, as discussed in the text, first-degree price discrimination is, in most cases, pie-in-the-sky, and not a reasonable aspiration for any industry, even a monopoly. P.I.E. as defined by the picture in Figure S15.8 is indeed the answer to the question, How much (dollar) value could this industry potentially produce for society? But the name Potential Industry Earnings gives a generally misleading idea of what is the industry's potential.

Indeed, suppose the good in question is transferable, so the industry can at best charge a single per-unit price. Suppose suppliers are weak, so the cost of inputs run along the marginal opportunity cost function. (Suppose the marginal opportunity cost function is flat, so we don't need to worry about firms in the industry paying more than the total social opportunity cost of inputs.) Suppose there are no prospects of entry. And suppose the firms in the industry, by hook or by crook, have effectively cartelized, setting their prices at the one price the industry would charge were it a monopoly. Then (a) the amount produced will be less than the socially efficient level, and (b) consumers will extract their consumer surplus above the "monopoly" price. From the perspective of P.I.E. and the Four Slices, this "loss" of P.I.E., is the result of customer power, namely the ability of customers to transfer the good once purchased, which forces linear prices on the industry rather than perfect price discrimination. Whatever Porter meant by customer power, I doubt he meant this.

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Proving the Efficiency of Competitive Markets

The text proves that a competitive market equilibrium maximizes total sur-

plus under fairly restrictive assumptions: Firms have rising marginal costs and no fixed costs, and consumers have utility functions of the form $u(x) + m$ for a concave u .

Proofs that work without these assumptions—that is, with somewhat weaker assumptions—are possible. I provide here a style of proof that is a good deal more general although a bit abstract.

Assume F firms produce the good in question, indexed by $f = 1, \dots, F$. Each firm is characterized by a total cost function $TC_f(z_f)$, where for this discussion, I use z_f to denote the amount of the good produced by firm f . I make no assumptions about these total cost functions.

Further assume I consumers can consume the good, indexed by $i = 1, \dots, I$. I cannot get away from assuming the consumers have *linear-in-money-left-over* utility functions of the form $v_i(x_i) + m_i$, where x_i is the amount of the good consumed by consumer i . (It is possible to deal with more general utility functions; see the following section.) I make no assumptions about the v_i functions.

I aim to prove the following: Suppose the economy is run by some mechanism: the market, a central planner, something. In the end, some total amount X of the good is produced. To produce X units of the good requires that the firms produce amounts z_f that sum to X . And the X units of the good must be split among the consumers. Letting x^i be the share of consumer i , we need that

$$\sum_i x_i = X = \sum_f z_f.$$

Monetary transfers also are required: I let t_i be the amount of money consumer i gives up to get his x_i units, and I'll let s_f be the amount of money firm f takes in. These monetary transfers must balance:

$$\sum_i t_i = \sum_f s_f.$$

Under this sort of arrangement, consumer i 's net benefit, measured in dollar terms, is $v_i(x_i) - t_i$, while firm f 's net benefit, also measured in dollars, is $s_f - TC_f(z_f)$. Hence, the total benefit of such an arrangement is

$$\sum_i [v_i(x_i) - t_i] + \sum_f [s_f - TC_f(z_f)].$$

But, since the dollar transfers balance, this is

$$\sum_i v_i(x_i) - \sum_f \text{TC}_f(z_f). \quad (\text{S15.1})$$

I assert that a competitive market equilibrium allocation for this good maximizes the expression S15.1, over all arrangements for production and consumption of the good.

By a competitive market equilibrium, I mean levels of demand and supply that arise from supply equals demand: There is a price p for the good, consumption levels x_i^* for the consumers, and production levels z_f^* for the firms, where for each i , x_i^* maximizes the consumer's net utility $v_i(x_i) - px_i$ at the price p , for each f , z_f^* maximizes the firm's profit $pz_f - \text{TC}_f(z_f)$ at the price p , and supply equals demand, or $\sum_i x_i^* = \sum_f z_f^*$.

I prove this as follows. Let \hat{x}_i and \hat{z}_f be some other plan for consumption and production of the good that satisfies feasibility: $\sum_i \hat{x}_i = \sum_f \hat{z}_f$. Note that there is no reason to suppose that this sum is the same as $\sum_i x_i^*$; this alternative plan could involve some other total amount produced and consumed. We know that, for each consumer i ,

$$v_i(x_i^*) - px_i^* \geq v_i(\hat{x}_i) - p\hat{x}_i,$$

since x_i^* is net utility maximizing at the price p for consumer i . And we know that, for each firm f ,

$$pz_f^* - \text{TC}_f(z_f^*) \geq p\hat{z}_f - \text{TC}_f(\hat{z}_f),$$

since z_f^* maximizes the profit of firm f at price p . So, if we sum all these inequalities, for all consumers i and all firms f , we get

$$\sum_i [v_i(x_i^*) - px_i^*] + \sum_f [pz_f^* - \text{TC}_f(z_f^*)] \geq \sum_i [v_i(\hat{x}_i) - p\hat{x}_i] + \sum_f [p\hat{z}_f - \text{TC}_f(\hat{z}_f)].$$

But, since $\sum_i x_i^* = \sum_f z_f^*$, all the "money" terms on the left-hand side sum to a net 0. And, since $\sum_i \hat{x}_i = \sum_f \hat{z}_f$, all the "money" terms on the right-hand side sum to a net 0. Therefore, the inequality just written simplifies to

$$\sum_i v_i(x_i^*) - \sum_f \text{TC}_f(z_f^*) \geq \sum_i v_i(\hat{x}_i) - \sum_f \text{TC}_f(\hat{z}_f).$$

The market-equilibrium plan does at least as well as the alternative. But this is true of any alternative, so the market-equilibrium plan maximizes the sum of the net benefits.

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Competitive Markets Are Production Efficient: or Allocating Production Among Cost-Independent Facilities

In the text, after proving (with some restrictive assumptions) that a competitive market equilibrium maximizes total surplus, this result was broken into three parts:

1. A competitive market equilibrium selects the right total quantity X to be produced and consumed.
2. A competitive market equilibrium arranges to have the total quantity X produced in a manner that minimizes the summed costs of the firms; it is production efficient.
3. A competitive market equilibrium arranges to have the total quantity X consumed in a manner that maximizes the total surplus generated from consumption; it is consumption efficient.

The manner of proof used in the preceding section is useful for proving these pieces separately. Let me illustrate with the second of these, production efficiency.

The question is the following: Suppose that, as commissar for the production of this good, you decide to produce X units in total. Your job as commissar for the production of the good is then to allocate this level of production among the F firms that can produce the good. You wish to do this to minimize the total cost of production. How do you do this?

If you have been reading all the supplementary material presented in this *Online Appendix*, you may recognize that we discussed (essentially) this problem at the end of the material for Chapter 11. There, instead of you having the role of commissar with F different firms, you were in charge of production allocation in a single firm, with some number, say F , cost-independent production facilities/processes. The commissar's job is to allocate production across the firms so that total production equals X and cost is minimized; back in the material for Chapter 11, the problem was to allocate production of X units in total to minimize cost. The political titles are different; the problem is the same.

In the material for Chapter 11, we assumed that the independent production facilities had no fixed costs and rising (or, at least, not-falling) marginal cost. Here I remove these assumptions and, even without them, I assert the following:

Let $s_f(p)$ be the competitive-market supply function of firm f ; that is, $s_f(p)$ is the solution to the problem of maximizing $pz - TC_f(z)$ for a fixed price p . Let $S(p)$ be the industry supply function, or $S(p) = \sum_f s_f(p)$. If the quantity X lies along the industry supply function (that is, if $X = S(p)$ for some price p), then the solution to the problem of procuring X units of the good as cheaply as possible is solved by having each firm produce its part of that supply; that is, have firm f produce $s_f(p)$ for the price p for which $X = S(p)$.

The proof is easy. Let \hat{z}_f be some other production plan for firm f , such that $\sum_f \hat{z}_f = X$. Because each firm f maximizes its profit at the price p by producing $s_f(p)$, we know that

$$ps_f(p) - TC_f(s_f(p)) \geq p\hat{z}_f - TC_f(\hat{z}_f).$$

Sum this inequality over all the firms, and you get

$$\sum_f [ps_f(p) - TC_f(s_f(p))] \geq \sum_f [p\hat{z}_f - TC_f(\hat{z}_f)].$$

This inequality can be rewritten

$$p \left(\sum_f s_f(p) \right) - \sum_f TC_f(s_f(p)) \geq p \left(\sum_f \hat{z}_f \right) - \sum_f TC_f(\hat{z}_f).$$

The first terms on each side of inequality are both pX , since $X = \sum_f s_f(p) = \sum_f \hat{z}_f$. They cancel out. Multiply the inequality by -1 , which flips the inequality sign, and you have precisely the inequality needed to verify the claim:

$$\sum_f TC_f(s_f(p)) \leq \sum_f TC_f(\hat{z}_f).$$

If X does not lie along the supply curve—which can happen if, for instance, supply jumps discontinuously at some price p ,¹ then things get a bit more complicated. But barring this unhappy possibility, it is clear what you as commissar—or as manager of cost-independent production facilities—should do. You should work out how much each firm/facility would produce to maximize profit at every price level p , taking that price level as given, form the horizontal sum of these firm/facility “supply functions,” and then see where this horizontal sum “hits” the desired quantity X . (In the material for Chapter 11, I closed with a cryptic remark about horizontal sums of marginal-cost functions. You now have everything you need to unravel any mystery that remark may have caused.)

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A Different Approach to the Invisible Hand: General Equilibrium

The method used in the previous two sections is also useful in proving that competitive markets are efficient in general. I italicized the s in markets because this subsection does something quite different from what we have done so far. So far, we have looked at a single market in isolation. In this section, I show what is known as a *general equilibrium approach* to market efficiency, where we consider all markets in the economy at once. Warning: This is going to take a lot of setting up, and it becomes very abstract. It has nothing to do with anything that follows in the book. So read this section only for the sheer pleasure of it, and if you find what follows pleasing, you might wish to reconsider your career goals and think about getting a Ph.D. in Economics.

The math that follows is not hard. But you must be comfortable with vector notation. Specifically, you must know that if p and x are two N -dimensional vectors, their “dot product,” written $p \cdot x$, is $p_1x_1 + p_2x_2 + \dots + p_Nx_N$.

Rather than talk about the market in one product or another, we talk about

¹ Individual supply, hence the sum of individual supplies, can only move “upward”: Suppose p and p' are two prices with $p > p'$, and suppose x is profit maximizing for the firm at price p and x' is profit maximizing at p' . Then $px - TC(x) \geq px' - TC(x')$ and $p'x' - TC(x') \geq p'x - TC(x)$. Add these two inequalities and, if you do a bit of algebra, you will discover that $(p - p')(x - x') \geq 0$. (If you’ve read this far, I might as well point out: This is true even if the price and output variables are vectors.) So if $p > p'$, it must be that $x \geq x'$. Now, x could equal x' : The rise in output price could result in no increase in supply. And, for a given price p , there could be more than one profit-maximizing output response; we see this in the text when there is a fixed cost: supply is either zero or efficient scale (and nothing between) when price is minimum AC. The “math” at work in these ideas is extremely general, but you’ll need to consult a doctoral level text—as always, my suggestion is Kreps, *Microeconomic Foundations I*—to see just how general.

the entire economy, all at once. In this economy, many products are made; let N be the number of products, and a vector $x = (x_1, \dots, x_N)$ is a bundle giving quantities of each of these products. The economy is composed of a large number H of consumers, indexed by $h = 1, \dots, H$, and a large number F of firms, indexed by $f = 1, \dots, F$.

- Each consumer begins life with an initial endowment of goods and a utility function as in Chapter 10. Because this is general equilibrium, money left over is not an argument of anyone's utility function; we assume that all the consumer's purchases are being considered simultaneously. The symbol h might seem funny for consumer, but the tradition in this part of economics is to refer to *households* instead of consumers.
- Each firm is characterized by a technology for turning inputs into outputs.

Money in this model is used purely as an accounting device. It has no residual value to anyone except that it represents purchasing power. Firms are owned by consumers; if a firm earns a positive profit, that profit is distributed to its owners in proportion their shareholdings and, once distributed, is used to purchase consumption goods.

We let x^h (h here is a superscript and not a power) denote the final consumption bundle chosen by consumer h . The production plan of firm f is denoted by z^f , where negative components of z^f represent inputs and positive components denote outputs. The initial endowment of consumer h is denoted by e^h . (All these are N -dimensional vectors.)

An *allocation* for this economy is a set of consumption bundles for all the households and production plans for all the firms. The allocation is *feasible* if each firm's production plan is technologically feasible for it and if the sum of goods consumed is less than the sum of initial endowments plus amounts produced (net of inputs to the production process), or

$$\sum_{n=1}^H x^h \leq \sum_{h=1}^H e^h + \sum_{f=1}^F z^f.$$

The test of efficiency in this setting is not the maximization of total surplus but instead what economists call *Pareto efficiency*. A feasible allocation for this economy is *Pareto efficient* if there is no other feasible allocation such that, when it comes to consumption, every consumer gets as much utility in the alternative as in the original allocation and some single consumer gets more utility. In other words, we cannot improve the utility of any consumer

without making someone else worse off. (Firm profits are not part of the measure of efficiency because profits are distributed to consumers; they are valuable only to the extent that consumers use them to improve their utility. We do not sum up consumer utilities because utilities are not measured on an equivalent “dollar-equals-utility” scale; utility functions are not of the money-left-over form.)

A market equilibrium for this economy is a feasible allocation (consumption and production plans) and a vector of nonnegative prices p for all the commodities such that (1) each firm is maximizing its profit at the prices p with its part of the allocation, and (2) each consumer is maximizing his or her utility, given prices p and given his or her resources, derived from the endowment the consumer starts with and profits paid out by the firms.

So far, no assumptions have been made about each firm’s technology or each consumer’s utility function. It is assumed that consumers maximize their utilities and that firms maximize profits. In addition, three other assumptions are made:

1. All consumers are *locally insatiable*. This is economic fancy talk for the property that each consumer, at each possible consumption bundle, can increase his or her utility if given a little more money to spend. If one of the goods always raises utility, that is more than adequate for this assumption. This assumption guarantees that a utility-maximizing consumer always spends all of his or her money. Since a consumer’s wealth consists of his or her endowment and profits received from firms, this in turn ensures that, in a market equilibrium, with prices p , consumption levels x^h , and firms’ production plans z^f , the market value of all the consumption bundles equals the market value of consumer endowments plus the sum of firms’ profits, or

$$\sum_h p \cdot x^h = \sum_h p \cdot e^h + \sum_f p \cdot z^f.$$

2. All markets are competitive or, in other words, all firms and all consumers are price takers.
3. Equilibrium prices are always nonnegative. This can be assumed directly or more primitive assumptions can be made that imply this. Examples of such more primitive assumptions are that at least one consumer has non-decreasing utility or at least one firm’s technology permits free disposal of any commodity.

The big result, which is so important to economists that it is known as *the first theorem of welfare economics*, is that the allocation part of a market equilibrium is efficient. (This is the invisible hand at work, in a general equilibrium. In case you are comparing this with a more advanced textbook, you should know that general equilibria are often referred to as *Walrasian equilibria*.)

It is remarkably easy to prove this result. Suppose prices p , consumption bundles x^h for $h = 1, \dots, H$, and production plans z^f for $f = 1, \dots, F$ constitute a market equilibrium. If the allocation part is not efficient, some other feasible allocation, given by consumption bundles \hat{x}^h and production plans \hat{z}^f , gives each consumer at least as much utility as the equilibrium allocation and gives one consumer more. Now, for the plan (\hat{x}^h, \hat{z}^f) to be feasible, we must have

$$\sum_h \hat{x}^h \leq \sum_h e^h + \sum_f \hat{z}^f.$$

Since prices are nonnegative, we can multiply each side of this inequality by prices and keep the inequality, or

$$\sum_h p \cdot \hat{x}^h \leq \sum_h p \cdot e^h + \sum_f p \cdot \hat{z}^f.$$

Firm by firm, since z^f maximizes profits at the prices p , $p \cdot z^f \geq p \cdot \hat{z}^f$. Therefore,

$$\sum_f p \cdot \hat{z}^f \leq \sum_f p \cdot z^f.$$

And consumer by consumer, $p \cdot \hat{x}^h \geq p \cdot x^h$. This is true because x^h maximizes the consumer's utility subject to his (or her) budget constraint at the prices p . But \hat{x}^h gives the consumer as much utility as x^h . If \hat{x}^h gives much utility and is cheaper than x^h , then (by local insatiability) the consumer could do better than x^h at prices p . Moreover, for any consumer (and there is at least one) for whom \hat{x}^h has higher utility than x^h , it must be that $p \cdot x^h < p \cdot \hat{x}^h$. Hence, summing over all consumers,

$$\sum_h p \cdot x^h < \sum_h p \cdot \hat{x}^h.$$

Now combine the three inequalities displayed in this paragraph:

$$\sum_h p \cdot x^h < \sum_h p \cdot \hat{x}^h \leq \sum_h p \cdot e^h + \sum_f p \cdot \hat{z}^f \leq \sum_h p \cdot e^h + \sum_f p \cdot z^f.$$

But, two paragraphs ago, we argued that the first and last terms in this progression must be equal. We have a contradiction to the assumption that the allocation part of a market equilibrium is not Pareto efficient.