# Chapter 17 Material

Solutions to the problems from Chapter 17 are provided, followed by a discussion of public goods.

**17.1** (a) If people distribute their drives so that the times are equal, the solution is found by solving

$$30 + \frac{n_B}{20{,}000} = 40 + \frac{n_T}{5000} \quad \text{and} \quad n_B + n_T = 400{,}000.$$

We can substitute $400{,}000 - n_B$ for $n_T$ in the first equation, getting

$$\begin{aligned}
30 + \frac{B}{20{,}000} &= 40 + \frac{400{,}000 - n_B}{5000} \quad \text{or} \\
\frac{n_B}{20{,}000} + \frac{n_B}{5000} &= 40 - 30 + \frac{400{,}000}{5000} = 90, \quad \text{which is} \\
\frac{5n_B}{20{,}000} &= 90, \quad \text{or} \quad n_B = 4000 \cdot 90 = 360{,}000,
\end{aligned}$$

which means that $n_T = 40{,}000$.

(b) The total commute time is

$$360{,}000 \cdot \left(30 + \frac{360{,}000}{20{,}000}\right) + 40{,}000 \cdot \left(40 + \frac{40{,}000}{5000}\right) = 400{,}000 \cdot 48,$$

which is 19.2 million minutes.

(c) The problem here is to minimize

$$n_B \cdot \left(30 + \frac{n_B}{20{,}000}\right) + n_T\left(40 + \frac{n_T}{5000}\right), \text{subject to } n_B + n_T = 400{,}000.$$

There are two ways to do this. (1) You can substitute $400{,}000 - n_B$ for $n_T$ in the objective (the thing to be minimized) and set its derivative (in $n_B$) to zero. (2) Or you can use the logic of Chapter 6 to say: At the optimum, the marginal impact of a bridge commuter on the total should equal the marginal impact of a tunnel commuter. That is,

$$30 + \frac{2n_B}{20{,}000} = 40 + \frac{2n_T}{5000},$$

and, of course, $n_B + n_T = 40{,}000$. Doing the latter (or the former), you get $n_T = 60{,}000$ and $n_B = 340{,}000$.

Note that this makes the commute times 47 for the bridge and 52 for the tunnel (instead of 48 for each). What is happening here is: On the margin, an extra commuter in the tunnel lengthens the trip for fewer people, so we want commute times in the tunnel to be longer.

(d) If we have $0 tolls, 360,000 commuters will take the bridge. We want to discourage bridge use down to 340,000 commuters, so we want to impose a toll on the bridge (but not the tunnel). (If we wanted to be revenue neutral, we could think of subsidizing tunnel travel somehow.) So suppose the toll on the bridge is $t_B$ (measured in dollars). The total "cost" to an individual commuter taking the bridge is then

$$10t_B + 30 + \frac{n_B}{20{,}000}$$

while for the tunnel is it

$$40 + \frac{n_T}{5000}.$$

The values of $n_B$ and $n_T = 400{,}000 - n_B$ will adjust so these are equalized, which is

$$\begin{aligned}10t_B + 30 + \frac{n_B}{20{,}000} &= 40 + \frac{400{,}000 - n_B}{5000}, \quad \text{or} \\ \frac{5n_B}{20{,}000} &= 40 + 80 - 30 - 10t_B = 90 - 10t_B, \quad \text{or} \\ n_B &= 4000(90 - 10t_B).\end{aligned}$$

We want $n_B$ in this equation to be 340,000, so we solve

$$340{,}000 = 360{,}000 - 40{,}000t_B,$$

which is $t_B = \$0.50$. A $0.50 toll on the bridge is the answer.

**17.2** (a) In part a, there is no possibility of entry or exit. Assuming $X$ is the total catch, the marginal cost for any fisherman is

$$\mathrm{MC}(x) = 10 + \frac{X}{1000} + \frac{2x}{100},$$

so supply by a single fisherman is where marginal cost equals price, or

$$p = 10 + \frac{X}{1000} + \frac{2s(p)}{100} \quad \text{or} \quad s(p) = 50\left(p - 10 - \frac{X}{1000}\right).$$

Total supply by 10 fishermen is

$$S(p) = 500\left(p - 10 - \frac{X}{1000}\right).$$

Supply equals demand is

$$500\left(p - 10 - \frac{X}{1000}\right) = 5000(60 - p),$$

or

$$p - 10 - \frac{X}{1000} = 10(60 - p) \quad \text{or} \quad 11p = 610 + \frac{X}{1000}.$$

But $X = 5000(60 - p)$, so this equation is

$$11p = 610 + 5(60 - p) \quad \text{or} \quad 16p = 910,$$

which is $p = \$56.875$, $X = 15{,}625$, and $x = 1562.5$.

(b) If there is free entry and exit from the market, the price falls until it equals the minimum average cost of fishermen. So first we find the efficient scale by equating marginal and average cost:

$$10 + \frac{X}{1000} + \frac{2x}{100} = \frac{10{,}000}{x} + 10 + \frac{X}{1000} + \frac{x}{100},$$

which gives 1000 for efficient scale and (therefore) minimum average cost (= MC(1000)) of

$$10 + \frac{X}{1000} + \frac{2 \times 1000}{100} = 30 + \frac{X}{1000}.$$

This must equal $p$, the equilibrium price. But, if $p$ is the equilibrium price, then $X = 5000(60 - p)$. So we have the equation that determines $p$:

$$30 + \frac{5000(60 - p)}{1000} = p \quad \text{or} \quad 330 = 6p \quad \text{or} \quad p = \$55.$$

This gives a total quantity of $5000(60 - 55) = 25{,}000$, which means 25 active fishermen.

Of course, each fisherman earns 0 profit. Consumer surplus is a triangle with height \$5 (per pound) and base 25,000 lbs, for the total consumer surplus of \$62,500.

(c) By imposing a tax of \$6 per lb of fish, both the marginal and average costs of fishermen are raised by \$6. This does not change the efficient scale of 1000, but it does change minimum average cost to

$$36 + \frac{X}{1000}.$$

This must be the new (post-tax) price, and to find $p$, we have to solve

$$36 + \frac{5000(60 - p)}{1000} = p \quad \text{or} \quad 336 = 6p \quad \text{or} \quad p = \$56.$$

This gives a total quantity of 20,000 lbs of fish and 20 active fishermen. Producer surplus is 0, and consumer surplus is $\frac{1}{2} \times \$4 \times 20{,}000 = \$40{,}000$. But net government revenues are $\$6 \times 20{,}000 = \$120{,}000$. So total surplus rises, and substantially, by virtue of this tax.

Because of the externality in fishing costs and the free entry assumption, it is not easy to find the inefficiency on a graph. But, in general, the cause of the free-market inefficiency is clear: Each fisherman, by raising the costs of others, exerts a negative externality. By taxing fish to depress the level of fishing, the externality is reduced.

What level of output maximizes social surplus? We first note that the efficient scale is 1000, regardless of industry scale. So, if we are going to produce $X$ units in total, it is most efficient to have $X/1000$ firms. Ignoring the problem of a noninteger number of firms, the total cost for providing $X$ most cheaply is

$$10{,}000\frac{X}{1000} + \left(10 + \frac{X}{1000}\right)X + \frac{X}{1000} \times \frac{1 \text{ million}}{100} = 10X + 10X + \frac{X^2}{1000} + 10X.$$

Hence, the true marginal cost is $30 + X/500$, and this equals marginal utility (which tracks inverse demand) where

$$30 + \frac{X}{500} = 60 - \frac{X}{5000} \quad \text{or} \quad 30 = \frac{11X}{5000} \quad \text{or} \quad X = 13{,}636.$$

Now, in fact, this does not quite work, because $X = 13{,}636$ implies 13.636 fishermen (if each catches 1000 lbs. of fish).

Since I'm unsure how we get 0.636 of a fisherman, if you want to restrict possible answers, you can safely assume that the answer is either 13 fishermen (probably catching a bit more than 1000 lbs. each) or 14 (catching a bit less than 1000 lbs. each). The easiest way to proceed at this point is to use Excel and Solver or similar; I'll leave it to you to do this, except to tell you that the answer is 14 fishermen and a total catch of 13,779 lbs of fish, or 984.21 lbs. of fish apiece.

**17.3** The firm should give each division the right to 90 units of service, but let them "trade these rights" among themselves in a central exchange, using transfer pricing in the determination of each division's profit that take into account these trades. For instance, the firm could ask each division: "You have 90 units of service allocated to you. You can sell some of them back to center if you wish to, and you can buy more units if you wish to. For each price per unit (say, in multiples of \$1000), would you want to buy more service or sell what you have, and how much. That is, how much would you want to buy/sell if the price per unit were \$1000? If the price were \$2000? And so forth." And, when it gets responses back, it looks for the price where the number of units the divisions want to sell equals the number that other dvisions want to buy.

And, to answer the final question, the "equilibrium price" the firm finds in this internal market is the so-called shadow value of a unit of service. If the firm can increase the capacity of the facility at a cost per unit below this shadow price, it should do so. If the marginal cost of a unit of the service is more than this shadow price, the firm should think about scaling back on the size of the service unit.

**17.4** When a car thief is looking for a car to try to steal and sees a "crowbar," he is probably going to move on to another car. That's why insurance companies will subsidize the use of "crowbars." With the LoJack device, the thief doesn't know that the device is there, but if he steals the car, it is very likely that the car will be recovered and quickly, so the insurance company is less likely to have to pay, even though the car was stolen.

The "crowbar" produces no externalities for other cars or, rather, it produces a slight negative externality: A thief deterred by the presence of a "crowbar" is simply going to look for another prospective victim.

But LoJack, to the extent that it means car thieves are more likely to be caught, deters car thieves generally. A car may not have LoJack, but the thief doesn't know this, and if the prospect of being caught deters the general activity of car theft, that's a positive externality generated by LoJack for all cars. (Of coure, the market penetration of LoJack has to be significant for this to happen.)

Note in this regard the following "conflict of interest." If your car has LoJack, you'd like to put on your car a visible sticker reading "This car protected by LoJack." If we assume that such stickers can only be put on cars that actually have LoJack, this deters the theft of those cars, but lowers—perhaps to zero—the positive externality of LoJack for other cars (without LoJack, hence without the sticker). The car owner with LoJack wants the sticker. Insurance companies other than his don't want him to have the sticker. And his insurance company is conflicted. But this presumes that only cars with LoJack would be able to have such a sticker, and while LoJack may be expensive, stickers—counterfeit stickers, perhaps—are cheap.

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## Public Goods

A public good is a good whose consumption by consumer A does not noticeably affect the ability of other consumers from consuming the same good. Clean air to breath, national defense, and lighthouses marking rocky shorelines are typically cited examples. In some cases, it is impossible for the provider of a public good to exclude everyone in the society from consuming the good. In other cases—the terminology used is *a public good with the possibility of exclusion*—the provider can pick and choose who does and does not get to consume the good. Setting aside the problem of congestion, the provision of national parks is the typically cited example of a public good with the possibility of exclusion (since the park can be fenced with an admissions gate set up).

We do not discuss public goods in the textbook chapter, but as they are often included as a topic in microeconomic textbooks, I'll say a few things about them here. To give you an introduction to the general remarks to follow, here is a problem that you can solve. (The solution follows.)

We are interested in a good provided in an economy consisting of 5 million individuals. Every individual in this economy has a linear-in-money-left-over utility function for this good, taking the form $v_i(x_i) + m_i$, where the subscript $i$ refers to the particular individual, $x_i$ is amount of this good consumed by $i$, and $m_i$ is the money $i$ has left over. Moreover, each $v_i$ takes the form $k_i \ln(x_i + 1)$ where $k_i$ is a constant specific to individual $i$: $k_i = 24$ for 1 million individuals; $k_i = 12$ for another 1 million; $k_i = 6$ for 1 million individuals; $k_i = 1$ for 1 million individuals; and $k_i = 0.5$ for the final 1 million individuals. The marginal cost of production of this good is a constant \$3.

(a) Suppose this is a private-consumption good, the sort analyzed and discussed in the book prior to this chapter. How much of the good should be produced, and how should it be divided among the 5 million consumers, in an efficient (surplus-maximizing) arrangement?

(b) Suppose for the remainder of the problem that this is a public good. This means that, if $X$ units of the good are produced in total, each consumer can consume $X$ units of the good, without affecting the consumption or utility gain of any other consumer. How much of the good should be produced in an efficient (surplus-maximizing) arrangement? (Since this is a public good, there is no issue of dividing the good among the individuals.)

(c) Suppose we provide this good using private contributions. That is, each individual in the economy decides on an amount to contribute, the contributions are summed, and if the total amount contributed is $C$, the amount provided is $X = C/3$. How much an individual chooses to contribute depends on how much he or she anticipates others contribute. Suppose, therefore, that one of the first million consumers believes that all the other individuals contribute nothing. In this instance, how much will this one individual contribute? And if she contributes that amount and the other 4,999,999 individuals anticipate that she will, how much does each of them contribute?

(d) Suppose the good is provided by the government: It levies a tax $t$ on each individual, raising \$5,000,000$t$, and it provides 5,000,000$t/3$ units of the good. If $t$ is set so that the government provides the socially optimal level of the good (your answer to part b) will any members in this society be worse off than if none of the good was provided?

(e) Suppose the good in question is a public good with the possibility of exclusion. The good is supplied by a monopolist, which announces that it will supply $X$ units of the good, and any citizen willing to pay \$$p$ can enjoy the good. What levels of $X$ and \$$p$ maximize the profit of this monopolist?

Here are the answers:

(a) If this is a private good, then the socially efficient outcome is where marginal utility (measured, with this sort of utility function, in dollars per unit) equals marginal cost. Since marginal cost is a constant \$3, this occurs for the first sort of individual, with utility function $24\ln(x_i+1)+m_i$, where $24/(x_i+1)=3$, or $x_i=7$. For individuals with utility function $12\ln(x_i+1)+m_i$, this is where $12/(x_i+1)=3$, or $x_i=3$. For individuals with the utility function $6\ln(x_i+1)+m_i$ this is where $6/(x_i+1)=3$, or $x_i=1$. And for individuals with the utility function $\ln(x_i+1)+m_i$ or $.5\ln(x_i+1)+m_i$, this is at $x_i=0$, since for these individuals, marginal utility at $x_i=0$ is 1 or 0.5, which is less than marginal cost.

So the answer is to produce 11 million units of the good, giving 7 to each of the 1 million folks with the first utility function, 3 to each of the second 1 million folks, 1 to each of the 1 million folks with $k_i=6$, and none to the last 2 million folks.

(b) If the good is a public good and if $X$ units are produced in total, then the total surplus generated is

$$\begin{gathered}1\text{ million}\times 24\ln(X+1)+1\text{ million}\times 12\ln(X+1)+1\text{ million}\times 6\ln(X+1)+\\ 1\text{ million}\times\ln(X+1)+1\text{ million}\times 0.5\ln(X+1)-3X=43{,}500{,}000\ \ln(X+1)-3.\end{gathered}$$

This is maximized where the derivative in $X$ equals 0, or where

$$43{,}500{,}000\left[\frac{1}{X+1}\right]-3=0,\quad\text{or}\quad X=14{,}499{,}999.$$

It is interesting that the total amount of the good produced is more here than would be the case if the good were a private good. But it is much more than that: In part a, individuals consume 0 or 1 or 3 or 7 units of the good. Here, each consumes almost 14.5 million units. If consumption of, say, Yosemite National Park were a matter of private consumption, individuals would consume very little "park." But efficiency leads to the conclusion that, if millions of folks can consume this in the manner of a public good—and I hasten to add that, with Yosemite Park, there is some congestion that makes it less than a pure public good—then a *lot* more park is consumed by each individual.

(c) If a person with utility function $24\ln(x_i+1)$ assumes that no one else makes any contribution, then she chooses the contribution $c_i$ to maximize $24\ln(c_i/3+1)-c_i$, which gives $c_i=\$21$. And if everyone else anticipates that she would contribute this amount, then each chooses a contribution $c_i$ to maximize

$$k_i \ln((21 + c_i)/3 + 1) - c_i,$$

where $k_i = 24, 12, 6, 1,$ or 0.5, depending on which of the five types of individual this person is. I let you do the checking, but with the constraint that the contribution $c_i$ must be nonnegative (an individual can't withdraw funds from the collection plate), the answer in every case is $c_i = 0$. Once the total contributions from any source are expected to be $21 or more, no one has a further incentive to contribute voluntarily. With public goods, the free-rider problem is ferocious.

(d) The person who obtains the least utility from this arrangement is a member of the fifth group, with utility function $0.5 \ln(X + 1) - m_i$. For $X =$ 14,499,999, this person's utility is ln(14,499,999 + 1) less the tax contribution required, which is $3 \times$ (14,499,999+ 1)/5,000,000 = \$8.70. Excel computes $0.5 \ln(14{,}500{,}000) = \$8.24$, so this person is worse off for being taxed to provide this public good.

(e) Suppose the monopolist produces $X$ units of the public good. It can then charge members of the first group up to $24 \ln(X+1)$ dollars for the privilege of consuming the good, members of the second group up to $12 \ln(X + 1)$, and so forth. It charges one of those five prices or a penny or two less; it has no reason to charge any less.

If it charges $24 \ln(X + 1)$, the net take will be

$$1 \text{ million} \times 24 \ln(X + 1) - 3X,$$

which is maximized in $X$ where

$$\frac{24{,}000{,}000}{X + 1} = 3 \quad \text{or} \quad X = 7{,}999{,}999.$$

If it charges $12 \ln(X + 1)$, 2 million citizens sign up, for a net take of

$$2 \text{ million} \times 12 \ln(X + 1) - 3X,$$

which is maximized in $X$ where

$$\frac{24{,}000{,}000}{X + 1} = 3 \quad \text{or} \quad X = 7{,}999{,}999.$$

If it charges $6\ln(X+1)$, 3 million citizens sign up, for a net take of

$$3 \text{ million} \times 6\ln(X+1) - 3X,$$

which is maximized in $X$ where

$$\frac{18{,}000{,}000}{X+1} = 3 \quad \text{or} \quad X = 5{,}999{,}999.$$

If it charges $\ln(X+1)$, 4 million citizens sign up, for a net take of

$$4 \text{ million} \times \ln(X+1) - 3X,$$

which is maximized in $X$ where

$$\frac{4{,}000{,}000}{X+1} = 3 \quad \text{or} \quad X = 1{,}333{,}332.33.$$

And if it charges $0.5\ln(X+1)$, all five million individuals sign up, for a net take of

$$5 \text{ million} \times 0.5\ln(X+1) - 3X,$$

which is maximized in X where

$$\frac{2{,}500{,}000}{X+1} = 3, \quad \text{or} \quad X = 833{,}332.33.$$

To find out which of these five gives the highest profit for the monopolist, we need to resort to a spreadsheet; Figure S17.1 shows such a spreadsheet. For each of the five "levels of service," we compute the price that would be charged, total revenue, and profit to the monopolist. We also compute the total amount of surplus generated by the arrangement. The numbers show that a monopolist is indifferent between serving only 1 million citizens or 2 million; the "tie" is because it gets twice the price with 1 million because the coefficients are 24 and 12, but twice as many people pay the lower price. In comparison with the efficient outcome, it serves fewer people (for a public

| | | D | E | F | G | H |
|---|---|---|---|---|---|---|
| 6 | Number of Citizens Who Sign | 1,000,000 | 2,000,000 | 3,000,000 | 4,000,000 | 5,000,000 |
| 7 | Level of X provided | 7,999,999 | 7,999,999 | 5,999,999 | 1,333,332.33 | 833,332.33 |
| 8 | Gross utility = Price to enter | $381.48 | $190.74 | $93.64 | $14.10 | $6.82 |
| 9 | Gross revenue | $381,478,850 | $381,478,850 | $280,930,860 | $56,412,771 | $34,082,972 |
| 10 | Cost | $23,999,997 | $23,999,997 | $17,999,997 | $3,999,997 | $2,499,997 |
| 12 | PROFIT | $357,478,853 | $357,478,853 | $262,930,863 | $52,412,774 | $31,582,976 |
| 13 | TOTAL SURPLUS PRODUCED | $357,478,853 | $548,218,279 | $637,505,344.14 | $602,437,286 | $590,543,724 |

*Figure S17.1. Problem 17.4(e): Finding the optimal price scheme for a public good with exclusion.* Five different "entry prices" can be tried, each the best for pursuing a subset of the entire citizenry. In this case, the first two, which go after the smallest sets of citizens, are tied for optimal.

good, it is always efficient to serve everyone) and restricts the output level as well.

## Discussion

With this problem and its solution as background, a few general remarks about public goods are in order. Throughout this discussion, I imagine a society with $N$ citizens, indexed $i = 1, \ldots, N$, who have linear-in-dollars-left-over style utility. Suppose that, if $Y$ units of a particular public good are produced and if exclusion is either impossible or not practiced, then the gross gain of utility of consumer $i$, measured on a dollars-left-over scale, is $u_i(Y)$. If consumer $i$ is excluded from the consumption of the public good, his gross gain in utility is $u_i(0)$. Therefore, if we charge consumer $i$ the amount $c_i$ for the public good and do not exclude him, his net benefit is $u_i(Y) - c_i$. To keep matters simple, I assume that it costs $kY$ to supply $Y$ units of the public good. Also suppose that $u_i(\cdot)$ is a nondecreasing function: No one prefers less of the public good to more, although we do not rule out the possibility that some people attach no (marginal) utility to more. (I also assume in what follows that $u_i(\cdot)$ is a concave function.)

## How Much Public Good to Provide: The Ideal

If the objective is to maximize the sum of the (dollar-measured) net consumer utility, how much of this public good should be provided? If we want to maximize the sum of net consumer utility, there would be no reason to exclude anyone; if $Y$ units are provided to a subset of the population, it costs nothing to extend this provision to everyone.

Hence, we want to find the value of $Y$ that maximizes $\sum_{i=1}^{N} u_i(Y) - kY$, or the sum of gross gains in utility less the dollar cost of providing the good.

To maximize this, set the derivative in $Y$ equal to 0, getting the optimality condition

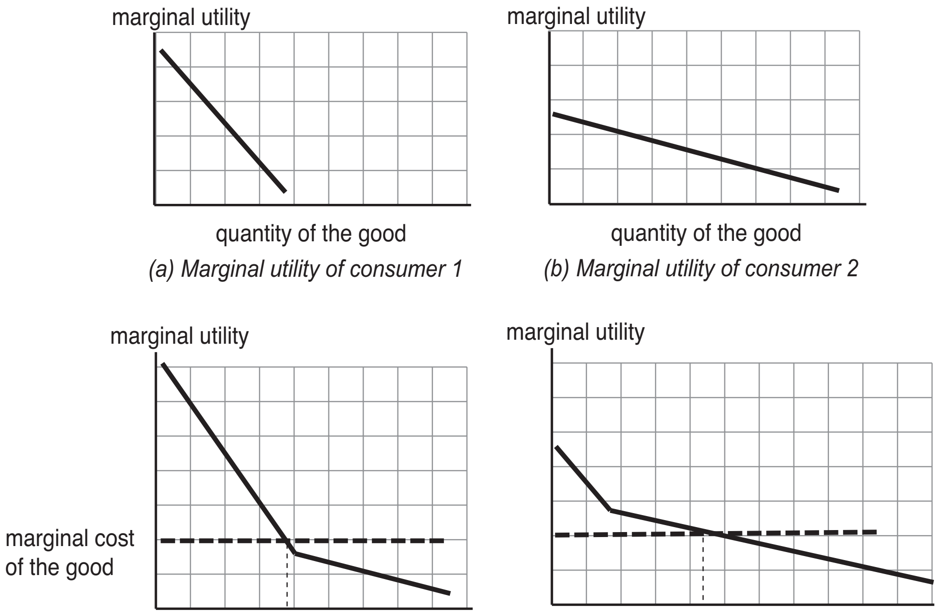
$$\sum_{i=1}^{n} \frac{\mathrm{d}u_i(Y)}{\mathrm{d}Y} = k.$$

In words, the sum of the marginal utilities for the public good, summed over all the citizens in the society, should equal the marginal cost of the public good.

Compare this with the level of private good provision that maximizes the sum of consumer surpluses. If $Y$ units of a private good are produced, at a cost to society of $c(Y)$, and those $Y$ units are divided among the citizens with citizen $i$ getting $y_i$ units of the good, the net gain to society is $\sum_{i=1}^{N} u_i(y_i) - kY = \sum_{i+1}^{N} u_i(y_i) - k(y_1 + \ldots + y_N)$. As we saw in Chapter 15, to maximize this we set $u_i'(y_i) = k$, or the marginal utility of each citizen's share of the private good should equal the marginal cost of producing all of the private good; hence, marginal utilities should be equal across citizens.

These are very different rules. To see how different they are, we can represent them graphically. We already know how to represent graphically the private-goods rule: We sum *horizontally* the marginal utility functions of each individual consumer, which is his or her inverse demand function, and see where this hits society's marginal-cost-of-provision function. But, with a public good, we want to equate marginal cost to the sum of the marginal utilities. So instead of summing individual marginal utilities horizontally, we have to sum them *vertically* (see Figure S17.2).

In the problem at the start of this section, consumers consumed more of the good if it were public than if it were private. Indeed, in the problem, if the good were a private good, the amount consumed by each consumer at the social optimum was on the order of a few units of the good, while the amount consumed by each consumer if the good is public was on the order of millions of units. And the total amount of the good produced if it were a public good was more than the total amount produced if the good were private. The first of these (that per capita consumption is more when the good is a public good than when it is private) is quite general, but the rest most certainly is not general. If you are unsure about this, consider the simple case in which all consumers are identical, with linear marginal utility $u_i'(y) = 10 - y$. Assume that $k$, the marginal cost of provision, is 1. Then, if the good is private, each consumer gets 9 units of the good at the efficient outcome. If there are 1 million citizens, 9 million units are produced. But if




*(a) Marginal utility of consumer 1*

*(b) Marginal utility of consumer 2*

*(c) For public goods, sum the marginal-utility curves vertically and find where the vertical sum intersects the marginal cost of provision of the good*

*(d) For private goods, sum the marginal utility curves horizontally and find the intersection*

*Figure S17.2. The efficient level of production of a private good and a public good, for two consumers.* In this case, more of the good is produced in total if it is a private good (panel d) than if it is a public good (panel c), but remember that the good must be divided between the two consumers if it is a private good; each consumer consumes the full amount of the good in panel c, while each gets only a share of the amount indicated in panel d. (In panel d, a bit more than 4 units of the good are provided. Slightly more than half of this goes to consumer 1. How do I know this?)

the good is a public good, then the amount produced never exceeds 10 units, no matter how many citizens there are, because at 10 units, every citizen's marginal utility for the good is 0.

## The free-rider problem and private provision of public goods

Public goods are typically provided by collectives, such as governments, rather than privately. The reason is the one we saw in part c of the problem: Public goods suffer from a very strong free-rider problem. Specifically, suppose funds for the public good are provided by voluntary subscription. If enough funds have been contributed to give $Y$ units of the public good, citizen $i$, deciding whether to contribute more for his own benefit, asks how $u_i'(Y)$ compares to $k$. Specifically, if $u_i'(Y) \leq k$, citizen $i$ has no desire to contribute any more funds, while if $u_i'(Y) > k$ and citizen $i$ believes that no one will contribute more, his incentives are to contribute up to the point where the funds collected are sufficient to finance $Y_i^*$ units, where $Y_i^*$ solves $u_i'(Y_i^*) = k$. In other words, for private reasons, citizens are ready to subscribe funds so that the total amount of the public good matches what they would choose for themselves if they were buying a private good for personal consumption.

This means that, once private contributions reach the level sufficient to finance the largest "private goods consumption level" (that is, $\max_i Y_i^*$), no one has any incentive to contribute more. Everyone wants a free ride on the contributions made by others.

So what does society do to get closer to the efficient level of public good provision? One possibility is to raise the marginal utility of donors for their donations. Societies can impose guilt on nongivers or provide esteem and public acclaim to those who make very large contributions. The first is a description of a norm of giving; the latter is why public benefactors get their names on buildings and other public goods they provide.

A second possibility, if the good is a public good with the possibility of exclusion, is to exclude those citizens who do not contribute an "entry" or "use" fee. Things might be arranged so that a private provider is given the right to provide this public good and exclude those who do not contribute. But, in such instances, society wants to be very careful in regulating the private provider. You saw some of this in part e of Problem 17.4; to learn more about this situation, an interesting exercise is to work through similar exercise if society consists of 1 million identical citizens, each with $u_i'(y_i) = 10 - y_i$, and if $k = 1$.

Most often, though, the government steps in, using its power to coerce contributions through the power to tax. In such circumstances, the government faces two basic problems:

First, the government must decide how much of the public good to provide. Interest-group politics makes this as much a political issue as an economic one, but suppose, taking a very idealistic perspective, that the government wishes to maximize total surplus. The government must try to ascertain the marginal values of each citizen for the public good at various levels of provision; then citizens have a positive incentive to misrepresent the value of the good to themselves.

Suppose the government goes to each citizen in turn and asks very directly What is your marginal utility for the public good at various possible levels of provision? The citizens understand that, in the end, the government will provide that level of the public good that equates the sum of the (reported) marginal utilities to the marginal cost of provision. Moreover, citizens understand that, to pay for the public good, some taxes will be imposed. To keep matters simple, citizens believe that they will be taxed equally for the public good. That is, $Y$ is the amount of the public good provided, so that $kY$ is the total cost of provision, then each citizen will be told to pay $kY/N$, where $N$ is the number of citizens.

It should be intuitive that, in this case, citizens who have a relatively high marginal value for the public good have the incentive to overstate that marginal value, while those who have a relatively low marginal value have an incentive to understate the value. I do not subject you to a mathematical derivation of this, but the idea is simple: Those who attach a relatively high marginal value to the public good want the government to supply more of it, even if they pay an equal share of the cost, because they realize that their fellow citizens will pay for most of the good. At the same time, those who attach a relatively low marginal value want less of it provided, given that they pay an equal share of the cost. In part d of Problem 17.4, you saw at least some of this, in that the folks who got the least utility from the public good came out behind, if the socially optimal level of public good were provided and equal taxation used to finance provision of the good. They, at least, would like to see less of the public good provided, so they would report lower marginal utilities.

Perhaps, you might think, the answer is to have citizens who say they want more of the good pay a higher share of the cost. It is harder to see, so I do not bother with a demonstration, but this does not work either. In fact, it can be proven that there is *no* way for the government to elicit honest responses to the question *What is your marginal utility for the public good?* if the government plans to use the information provided to determine the efficient level of provision, with some form of taxation imposed to pay the cost. To prove this is quite difficult (see, for example, Section 18.3 in D. M. Kreps, *A Course in Microeconomic Theory*, Princeton, NJ: Princeton University Press, 1990). But common sense probably suggests that no matter what mechanism is used to try to elicit this information, someone in the population will try to take advantage of the elicitation scheme to influence unduly the social decision.

Despite this, government economists often engage in benefit–cost analyses in an attempt to measure the benefits and costs to society of certain types of public goods such as roads or dams for flood control. Decisions on how much of these goods to provide are rarely determined solely by these studies (political considerations play a rather large role), but a good deal of attention is paid to how best to estimate the social benefits and costs of public goods, as an aid in determining how much to provide. If you are interested in public sector economics, you will probably meet this topic again.

## Paying for the Public Goods

Once it is determined how much of a public good to provide, funds must be found to pay for it. Governments usually rely on general tax revenues, such as revenues from income or sales or value-added or property taxes, to pay for these things.

In Chapters 15 and 16, many things are said, none of them very nice, about government interference in perfectly competitive markets. Every time the government places a tax on the good in the market, we wind up with a deadweight cost. In the calculations of those chapters, we valued \$1 of consumer surplus equal to \$1 of producer surplus equal to \$1 of government net revenue. Thus, financial transfers from a tax were a net wash; all that matters to efficiency is the impact on physical goods outcomes.

In cases of public goods, the story is not so simple. Suppose we decide to have public provision of a particular public good. The government needs to raise money to provide the good. One way is to impose a tax on the sale of some other good. This may cause a deadweight cost in the market for the second, taxed good, but the amount of this deadweight cost may be less than the gain in value that comes from the provision of the public good. This happens because, when government net receipts are used to provide public goods, \$1 in a consumer's or producer's pocket can generate less surplus than \$1 of government net receipts. In essence, \$1 spent on public goods can generate a lot more benefits than that \$1 spent on "private goods," which is where it goes if it winds up in the pockets of a producer or consumer. So a good reason for governments to impose taxes, even on goods for which there are no externalities, is to raise the funds needed to pay for public goods. In such a case, the government may wish to tax goods for which the deadweight cost is particularly small, with a very inelastic supply or demand, but still, when we consider where the money would go, even a moderate deadweight cost for the good may be a worthwhile sacrifice.

(If we could attach a deadweight price tag to revenue raised to pay for the public good, we could include that cost in the cost of provision of the public good when determining how much of the public good to provide. That is, the socially optimal level of public good is no longer where the sum of marginal utilities equals the direct marginal cost of the public good but where the sum of marginal utilities equals the direct marginal cost plus the marginal deadweight cost.)

For more on public goods and related topics, seek out a good textbook on public finance and public sector economics.