Chapter 18 Material

Solutions to the problems from Chapter 18 are provided. Please note that Appendix 4, which follows, discusses the normative application of the expected-utility model.

18.1 See Table S18.1. Note that the utility levels for Jo and, from her expected utility (EU) levels, her certainty equivalents (CE), are read by hand off the graph. For Kreps and Patel, we can use the formulas on page 410 of the text. In particular, Kreps' certainty equivalent for Gamble B is $-\ln(-CE)/0.0001$.

Jo			utility	EU	CE
	Prizes	Probabilities			
Gamble A	$5,000	1.00	0.7	0.7	$5,000
Gamble B	$40,000	0.40	1	0.4	-$4,500
	-$10,000	0.60	0		
Gamble C	$21,000	0.33	0.9	0.593	$1,000
	$9,000	0.33	0.78		
	-$9,000	0.33	0.1		
Kreps			utility	EU	CE
	Prizes	Probabilities			
Gamble A	$5,000	1.00	-0.95	-0.951	$5,000
Gamble B	$40,000	0.40	-0.67	-0.931	$7,125
	-$10,000	0.60	-1.11		
Gamble C	$21,000	0.33	-0.81	-0.94	$6,234
	$9,000	0.33	-0.91		
	-$9,000	0.33	-1.09		
Patel			utility	EU	CE
	Prizes	Probabilities			
Gamble A	$5,000	1.00	-0.9	-0.905	$5,000
Gamble B	$40,000	0.40	-0.45	-0.913	$4,574
	-$10,000	0.60	-1.22		
Gamble C	$21,000	0.33	-0.66	-0.897	$5,462
	$9,000	0.33	-0.84		
	-$9,000	0.33	-1.2		

Table S18.1. The solution to Problem 18.1.

Also note that I don't really need to compute EU for Gamble A, if all I'm interested in is the CE of this gamble: Since Gamble A is a sure $5000, $5000 is its CE.

Of the three, Jo definitely prefers Gamble A, Kreps is going for Gamble B, and Patel is going for Gamble C, although for Patel, Gamble C is worth only $462 more than Gamble A. (Since Patel has constant risk averse utility, that last statement is exact.)

18.2 See Table S18.2. Note that before I compute utilities for these three, I have to convert the prize level to their bank account balance. And when, after computing EUs, I want the CEs, I can express the CE as the certainty-equivalent bank-account balance or as the certainty-equivalent prize.

Alice	Prizes	Account balance	Probabilities	utility	EU	CE account balance	CE prize
Gamble A	$5,000	$55,000	1.00	234.52	234.5	$55,000	$5,000
Gamble B	$40,000	$90,000	0.40	300.00	240	$57,600	$7,600
	-$10,000	$40,000	0.60	200.00			
Gamble C	$21,000	$71,000	0.33	266.46	237.3	$56,302	$6,302
	$9,000	$59,000	0.33	242.90			
	-$9,000	$41,000	0.33	202.48			

Bob	Prizes	Account balance	Probabilities	utility	EU	CE account balance	CE prize
Gamble A	$5,000	$20,000	1.00	141.42	141.4	$20,000	$5,000
Gamble B	$40,000	$55,000	0.40	234.52	136.2	$18,560	$3,560
	-$10,000	$5,000	0.60	70.71			
Gamble C	$21,000	$36,000	0.33	189.74	140.7	$19,798	$4,798
	$9,000	$24,000	0.33	154.92			
	-$9,000	$6,000	0.33	77.46			

Bill	Prizes	Account balance	Probabilities	utility	EU	CE account balance	CE prize
Gamble A	$5,000	$55,000	1.00	10.92	10.92	$55,000	$5,000
Gamble B	$40,000	$90,000	0.40	11.41	10.92	$55,326	$5,326
	-$10,000	$40,000	0.60	10.60			
Gamble C	$21,000	$71,000	0.33	11.17	10.93	$55,586	$5,586
	$9,000	$59,000	0.33	10.99			
	-$9,000	$41,000	0.33	10.62			

Table S18.2. The solution to Problem 18.2.

In this case, Alice and Bob have the same utility function for their bank account balance, but Bob starts with less money in the bank, so (for this utility function) Bob is more risk averse. While Alice chooses Gamble B, with Gamble C in second place, Bob reverses the order, with Gamble A best. Bill and Alice have the same starting bank account balance, but since Bill is more risk averse than Alice (see the second bullet point on page 415 of the text), he opts for still-risky-but-not-most-risky Gamble C.

18.3 (a) For Jack, the gamble gives him a bank balance of \$45,000 with probability 1/4 and \$5000 with probability 3/4. This gives him an expected utility of

$$(0.25)\sqrt{45000} + (0.75)\sqrt{5000} = 106.066.$$

If he instead takes \$7500 for sure, his bank account balance will be \$12,500, for an (expected) utility of $\sqrt{12500} = 111.8034$. Jack prefers the sure thing.

(Although the problem doesn't ask for this, you can compute a certainty-equivalent *bank balance* for Jack if he takes the gamble by squaring the expected utility level of 106.066: $106.66^2 = \$11{,}250$. Since the sure \$7500 gives him a bank balance of \$12,500 for sure, he prefers the sure thing.)

And for Jim, taking the gamble gives him an expected utility of

$$(0.25)\sqrt{90{,}000} + (0.75)\sqrt{50{,}000} = 242.705,$$

while \$7500 gives him a bank balance of \$57,500 which has utility $\sqrt{57{,}500} = 239.79$, so Jim prefers the gamble. (And Jim's CE bank balance from the gamble is \$58,905.765.)

(b) If Jim must pay \$7500 to take the bet, his expected utility is

$$(0.25)\sqrt{82{,}500} + (0.75)\sqrt{42{,}500} = 226.424,$$

which has a CE bank balance of 226.424^2 =\$51,267.60, more or less. So if the alternative is not betting at all, Jim would be happy to pay \$7500.

(Note that, if it is a choice between the bet for free and \$7500 for sure, he takes the bet, and the difference between the CE bank balance and \$57,500 in the bank is \$1405.765, a bigger difference than between is CE bank balance if he gambles for a fee of \$7500 or sticks with his current balance of \$50,000. Jim—and for that matter Jack—are less risk averse the wealthier they become.)

So how much will Jim pay for the right to take this gamble? If asked to pay \$7500, we know he is \$1267.60 to the good, so a first guess might be \$8767.60.

But if you compute his expected utility if he is asked to pay this much you get an expected utility of

$$0.25\sqrt{81{,}232.40} + -.75\sqrt{41{,}232.40} = 223.546,$$

hence a CE bank balance of $223.546^2 = \$49{,}973$. So \$8767.60 is too much. We need to find the number F (for fee) that makes

$$0.25\sqrt{90{,}000 - F} + 0.75\sqrt{50{,}000 - F} = \sqrt{50{,}000}.$$

There is no way to solve this analytically—it must be done numerically—and on Excel, I came up with $F = \$8741.20$ as pretty close.

(c) For Jack, since he only has \$5000 in the bank, if we ask him to pay anything more than \$5000, we have to specify what this means. If we want him to pay "cash up front" and he can't borrow the extra from Jim or someone else, he is stuck. If we say "pay later" and he is allowed to declare bankrupcty and only pay us \$5000 if he doesn't win the \$40,000, we get another answer.

So I won't go further with the question, Will he pay an amount more than \$5000? But we can answer, will he be willing to pay \$5000. That is, for a 1/4 chance at a bank account of balance of \$40,000, is he willing to risk (with probability 3/4) a bank account balance of \$0. In terms of expected utilities, this is the question, is

$$0.25\sqrt{40{,}000} + 0.75\sqrt{0} \quad \text{bigger than} \quad \sqrt{5000}\,?$$

This we can answer analytically, since the $\sqrt{0}$ term is 0: we want to know whether $0.25\sqrt{40{,}000} > \sqrt{5000}$? Square both sides, and you get $35{,}000/16 = 2500$, which is a lot less than 5000. So now we need to find the biggest fee he will pay, which is the solution to

$$0.25\sqrt{45{,}000 - F} + 0.75\sqrt{5000 - F} = \sqrt{5000};$$

I get $F = \$4270.51$ as pretty close to precisely the answer.

18.4 (a) The insurance company takes in \$40,000 in premium from this policy, and its expected payout is $0.05 \times \$750{,}000 = \$37{,}500$, so its expected net from the policy is a gain of \$2,500.

(b) If the individual is risk neutral, then without insurance he "owns" a gamble with prizes \$1 million with probability 0.95 and \$250,000 with probability

0.05, for an expected value of $962,500. If he buys insurance, he will have a net asset position of $960,000 regardless of whether there is a fire or not. So he is better off without insurance.

(c) But if the individual is a risk-averse expected utility maximizer with the utility function $\sqrt{x}$ where x is his net asset position, then with insurance, he has a net asset position of $960,000 with certainty. Without insurance, he "owns" the lottery shown in Figure S18.1, where I supply the utility levels and compute expected utility. His expected utility is [975]. We can either find the utility of $960,000 for sure (by evaluating $\sqrt{960{,}000} = [979.796]$) or, a bit more informative, we can find his certainty equivalent for the utility level [975] by squaring 975 (since utility is the square-root of the dollar value): $975^2 = \$950{,}625$, and the decision maker is about $10,000 better off with insurance than without.

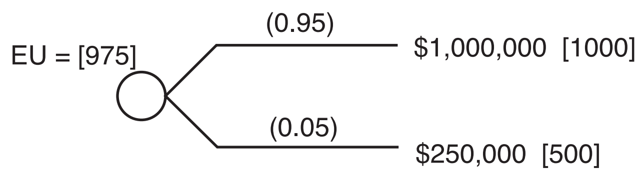

Figure S18.1. Problem 18.4: The lottery facing the decision maker if he does not buy insurance.

(d) To evaluate the possibility of fractional insurance, I built a simple spreadsheet, which produced the table shown as Table S18.3. Most of the spreadsheet should be self-explanatory; the key rows are the calculation of the net position of the individual if he has no loss and if not, depending on what fractional insurance he buys (as specified in the fifth row). These net positions are then converted into utility levels, and expected utility and certainty equivalents are computed.

	full insurance	no insurance	20%	40%	60%	80%	83.6%
premium	$40,000	$40,000	$40,000	$40,000	$40,000	$40,000	$40,000
prob loss	0.05	0.05	0.05	0.05	0.05	0.05	0.05
asset position no loss	$1,000,000	$1,000,000	$1,000,000	$1,000,000	$1,000,000	$1,000,000	$1,000,000
amount of loss	$750,000	$750,000	$750,000	$750,000	$750,000	$750,000	$750,000
fraction insurance	100.00%	0.00%	20.00%	40.00%	60.00%	80.00%	83.60%
net if no loss	$960,000	$1,000,000	$992,000	$984,000	$976,000	$968,000	$966,560
net if loss	$960,000	$250,000	$392,000	$534,000	$676,000	$818,000	$843,560
utility if no loss	979.80	1,000.00	995.99	991.97	987.93	983.87	983.14
utility if loss	979.80	500.00	626.10	730.75	822.19	904.43	918.46
expected utility	979.7959	975	977.49732	978.90701	979.64038	979.89809	979.9037
CE	$960,000	$950,625	$955,501	$958,259	$959,695	$960,200	$960,211

Table S18.3. Problem 18.4(d): Fractional insurance.

I began with the extreme cases of 100% and no insurance, verifying the calculations already done. Then I tried (in successive columns) 20%, 40%, 60%, and 80% insurance, finding that the individual's CE position kept improving with more and more insurance, although at 80% he is better off than at 100%. Finally, in the last column, I asked Solver to maximize the cell with the CE in it varying the fraction of insurance; Solver declared that 83.6% insurance was best, for a CE of $960,211, a full $211 (in CE) better than full insurance. (Asking Solver to maximize the entry in the cell that gives EU would give the same result.)

18.5 (a) If Professor Patel begins with $500,000 in assets, the CE (in terms of final assets) of Gamble A is, of course, $550,000. Finding the EUs and CEs of Gambles B and C are easy to do in Excel (once you get the hang of working with exponential utility functions), so I'll just report the answers: The CE asset position of Gamble B is $558,839 and of Gamble C is $558,106. Professor Patel prefers Gamble B.

(b & c) Since Professor Patel's utility function is a constant-risk-aversion utility function, shifting his initial asset position simply shifts his CE-net assets for each gamble by the same amount. Take Gamble B, for instance: If his CE in final net assets for Gamble B when he has $500,000 in starting assets is $558,839, then if he begins with $1 million in assets, his CE for Gamble B in final net assets is $1,058,839, and if he begins with $0 in assets, his CE for Gamble B in final net assets is $58,839. Of course, this means that his preference for Gamble B doesn't change as we change his initial asset position.

You can verify all this using Excel, if you wish to.

(d) Professor K. Patel, in terms of risk preferences, is *identical* to his brother. Multiplying this sort of utility function by a positive constant and / or adding a constant just shifts EU in the same way and has no impact on CEs.

Once again, you can verify all this with Excel.

18.6 This problem is answered in detail in Section 19.1, so look there for the analysis. But the answers are: The CE for Jan for the whole gamble is $-\$1000$, more or less. She won't take it. But if she divides it up into 100 shares and finds folks with the same utility function that she has, they would value their shares at around $123.516 and so, since they have constant-risk-aversion utility functions, would be more than happy to pay $100 per share.