

Chapter 2 Material

The problems from Chapter 2, excluding Problem 2.7, from the text are solved, together with discussion of the case-let that begins on page 43 of the text. The solution to Problem 2.7 is found in Appendix 3, which follows the solution of problems from Chapter 3.

2.1 (a) Row 1, Column 2 is not a Nash equilibrium. While column 2 is a best response to row 1, row 2 is a better response to column 2 than is row 1.

(b) If Row chooses row 1, Column does best with column 2, to which row 1 is a best response. *So Row 1, Column 2 is a Nash equilibrium.*

If Row chooses row 2, Column best-responds with column 1, to which row 2 is a best response. *So Row 2, Column 1 is a (not) Nash equilibrium.*

If Row chooses row 3, Column's best response is column 3, to which row 3 is a best response. *So Row 3, Column 3 is a (third) Nash equilibrium.*

2.2 Against row 1, columns 1 and 2 are tied for best, but against both of those, row 1 is not best. No Nash equilibrium involving row 1.

Against row 2, column 2 is best, against which row 2 is best. *Row 2, Column 2 is a Nash equilibrium.*

Against row 3, column 1 is best, against which row 2 is best. So no Nash equilibrium involving row 3.

Against row 4, column 3 is best, against which row 4 is best. *Row 4, column 3 is a Nash equilibrium.*

2.3 Column 4 strictly dominates column 2, so column 2 is out. Once column 2 is out, row 1 strictly dominates row 3, so row 3 is (iteratively) out. Then column 3 dominates column 1, so column 1 is out. Then row 1 dominates row 2, and once row 2 is gone, column 4 dominates column 3. So by iterated application of strict dominance, the game is *dominance solvable*, with (only) row 1 and column 4 remaining.

Although you weren't asked to comment on this, *when a game is dominance solvable using (at each step) strict (iterated) dominance, what remains is the unique*

Nash equilibrium of the game. More generally, any strategy eliminated by iterated strict dominance cannot be part of any Nash equilibrium.

2.4 There is no immediate strict dominance of any row or column.

But columns 1 and 2 weakly dominate column 4, and row 1 weakly dominates row 3.

So I'll remove column 4 and row 3. But, that's all I can do.

2.5 (a) If you bid \$2000 in a first-price auction, and you win the auction, you will net \$0; you will be paying exactly what the trip is worth to you. And if you lose the auction, you will net \$0. So no matter what other bidders do, your outcome if you bid \$2000 is \$0. But if you bid \$1950, and if the highest bid other than your own is less than \$1950, you will net \$50 in value; if the highest bid other than your own is more than \$1950, you net \$0. So in some circumstances (for some "moves" by the other players) you do better than by bidding \$2000, and in (all) other cases you do just as well. Hence bidding \$1950 weakly dominates bidding \$2000.

But we cannot compare on the basis of any form of dominance bidding \$1950 versus bidding \$1960. Bidding \$1950 is better if the highest bid other than your own is less than \$1950, since with \$1950 your net is \$50 versus \$40 by bidding \$1960. But if the highest bid other than your own is, say, \$1955, bidding \$1960 gets you \$40 in value, while \$1950 gets you \$0.

Finally, bidding \$2000—which we already know gives you \$0 no matter what any other bidders do—weakly dominates bidding more than \$2000: If you bid more than \$2000 and lose the auction, you get the same \$0 you get by bidding \$2000. But if you win the auction, by bidding more than \$2000 you lose value on net.

(b) Compare bidding \$2000 with any bid over \$2000, say \$2050. Take cases:

- The highest bid other than your own is above \$2050. Then your net is \$0 in either case.
- The highest bid other than your own is below \$2000. Then you pay that highest bid and win the vacation in either case, so the two are tied.
- The highest bid other than your own is between \$2000 and \$2050. Then you lose the auction and net \$0 if you bid \$2000, and you win the auction and pay more than \$2000, which is more than the trip is worth to you, if you bid \$2050. You are better off bidding \$2000.

And compare bidding \$2000 and some amount less than \$2000, say, \$1950.

- If the highest bid other than your own is above \$2000, you lose the auction with either bid, so you net \$0 in either case.
- If the highest bid other than your own is below \$1950, you win the auction and pay the second highest bid (including than your own) in either case. So you do just as well with either bid.
- If the highest bid other than your own is between \$1950 and \$2000, then by bidding \$1950, you lose the auction and net \$0. By bidding \$2000, you win the auction and pay less than \$2000 for a vacation worth \$2000 to you. So you are better off bidding \$2000 than bidding \$1950.

(To be very precise, we should cover the cases where the highest bid other than your own is tied with your bid. For those you need a tie-breaking rule, which the problem did not specify. But for any reasonable tie-breaking rule, the sorts of arguments just given still work.)

2.6 In the threat game, if A is challenged, she will acquiesce. So B chooses between challenging and getting 1, and not challenging and getting 0. B will challenge A, and A will acquiesce.

And in the trust game, if B trusts A, A will act abusively ($2 > 1$). So B must choose between trust, getting -1 for himself, and no trust, which nets him 0. He will not trust A, and A will have no opportunity to move.

These two games are the basis of a lot of discussion in Chapter 4 in the text, concerning credibility and reputation, so I'll leave you to read about them there.

Problem 2.7 is solved in Appendix 3, following the material for Chapter 3.

Analysis of the Case-let

I reproduce both Figures 2.5 and 2.6 from the text. My assertion is that Figure 2.6 is the strategic-form representation of the "game" between A and B depicted in Figure 2.5, which is where A does indeed move first (and B sees whether A entered but not whether A got the cost info before B must make its entry decision.) Here is one of the calculations that must be done to show that this is so:

In Figure 2.5, look at the branch where A does R&D, learns it is high cost, enters, and B does not enter. (This is the branch furthest to the north-east in

the figure.) B's payoff is 0, of course, and A gets the market all to itself, for a gross \$30, but it costs an additional \$5 for the info, so A's net is \$25. This is the endpoint valuation shown.

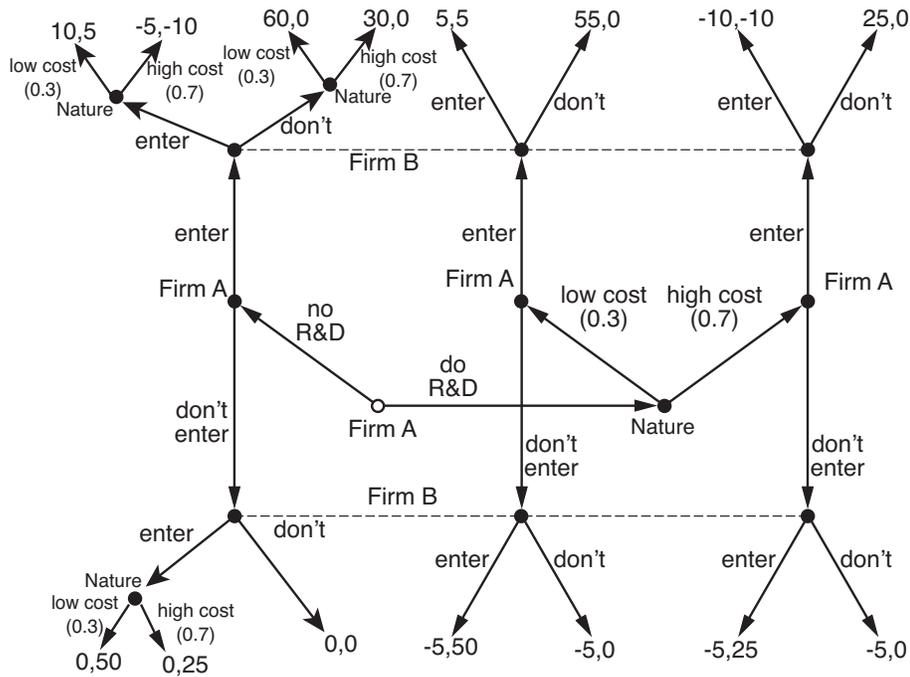


Figure 2.5. The game between A and B if A moves first, in extensive form.

Firm A's strategy:	Firm B's strategy:			
	Enter regardless of what A does	Enter if A enters, not if not	Enter if A doesn't enter, not if A enters	Don't enter regardless
Don't do R&D, enter	-0.5, -5.5	-0.5, -5.5	39, 0	39, 0
Don't do R&D, don't enter	0, 32.5	0, 0	0, 32.5	0, 0
Do R&D, enter regardless	-5.5, -5.5	-5.5, -5.5	34, 0	34, 0
Do R&D, enter if costs low (only)	-2, 19	-2, 1.5	13, 17.5	13, 0
Do R&D, enter if costs high (only)	-8.5, 8	-8.5, -7	16, 15	16, 0
Do R&D, don't enter regardless	-5, 32.5	-5, 0	-5, 32.5	-5, 0

Figure 2.6. The game between A and B in strategic form.

Turning to Figure 2.6, in the text I worked through the strategy profile where A does R&D and enters only if costs are low, while B enters regardless of what A does. Let me do another cell here: A doesn't do R&D and enters, while B enters if A doesn't enter and stays out if A enters. (This is row 1, column 3 in Figure 2.6.)

For this pair of strategies (or strategy profile), the outcome is that A enters and B, seeing A entering, does not. In Figure 2.5, we are looking at the second two outcomes in the northwest quadrant of the tree. B gets 0, of course, while A is looking at 60 with probability 0.3 and 30 with probability 0.7, for an expected outcome of $(30)(0.7) + (60)(0.3) = 21 + 18 = 39$. Which is what is in that cell in Figure 2.6.

So we have the situation modeled both ways. Now, what do these models tell us?

We said when discussing A's strategies that if A was going to enter no matter what the information said, it was silly to get the information: Why pay 5 for information that you won't use? Similarly, it is silly for A to pay 5 for information if A plans not to use the information and just stay out. You can see this in Figure 2.5 if you look hard enough (and know where to look). But this is easy to see from Figure 2.6:

- Row 1 strictly dominates row 3—the difference in strategy is that row 1 is enter without the info, while 3 is first get the info and then enter regardless.
- Row 2 strictly dominates row 6, on similar grounds.

But be careful of this logic. This works because B doesn't see whether A decided to get the info. *If B sees whether A gets the info or not, it can affect B's decision whether to enter. In particular, if B is playing a strategy where it doesn't enter if it sees that A does get the information but it enters otherwise, even if A plans to ignore the information, it might be worth 5 to keep B out. A might want to reformulate the model where B sees whether A gets the info, to know how that variation plays out.*

We can also see that rows 4 and 5 are strictly dominated by row 1: Although this is harder to tell without doing the modeling, the gain from being informed about costs isn't worth the price of the information. Going back to Figure 2.5, the logic is: If A gets the information that costs are high and doesn't enter, it has spent 5. If it doesn't get this information and just enters, the worst that happens to it is that B enters and it gets -5 when costs are high. By getting the information, it might avoid this bad outcome. But this bad outcome only has probability 0.7. Better to have a 70% chance of losing 5 than spending 5 for sure.

(You might expect that row 4 would dominate row 5: If you get the information and use it, aren't you better off entering when the information is

positive [costs are low] than when it is negative [costs are high]? This works for the first two columns. But row 5 is better than row 4 in columns 3 and 4. This happens because, in columns 3 and 4, Firm B is staying out when Firm A enters. It is true that firm A makes higher profits when it is in the market alone and costs are low than when it is in the market and costs are high: its profit gross of the costs of the information is 60 in the first case and 30 in the second. [The cost of the test is a sunk \$5 in either case and so is irrelevant to this comparison.] But costs are low only 30% of the time. And it is better to get 30 with probability 0.7 [expected value 21] than 60 with probability 0.3 [expected value 18]. Note that the numbers in the cells of Figure 2.5 in these row–column combinations are 16 and 13, net of the costs of the test.)

So we've eliminated rows 3, 4, 5, and 6 by strict dominance: A is not going to get the information. The only choice left is whether to enter or not.

But now we have a game without information sets—it is the game depicted in Figure S2.1.

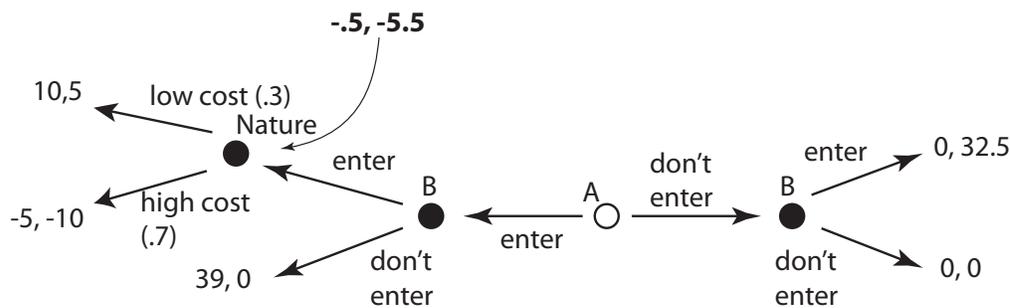


Figure S2.1. The entry game with many strategies pruned.

Firm A decides whether to enter or not. Firm B, seeing what A does, decides to enter or not. Then there should be a node for nature's choice—will it be high cost or low?—but I've included this node only for the case where they both enter, for the following reason: Once you have Nature's node, since it comes last, you compute expected values for each of the two firms. So, in the one place this node is included, A gets 10 with probability 0.3 and -5 with probability 0.7, for an expected value of -0.5 . And B gets 5 with probability 0.3 and -10 with probability 0.7, for an expected value of -5.5 . (Note that I've indicated those two expected values in the picture.) If I had done the same thing for the three other places there should be a move by nature, I'd have the payoffs indicated at those spots.

So, what does this tell us? If A enters, B can choose to enter—expected value -5.5 —or stay out, worth 0. B will stay out. So if A enters, it nets an expected 39. And if A doesn't enter, B will do so; A gets 0. So, if A moves first, A

should enter; this will keep B out of the market (A has pre-empted B), and A wins.

We could have seen this by continuing our dominance analysis in Figure 2.6: Once only rows 1 and 2 remain, column 1 dominates column 2 (weakly), and column 3 dominates column 4 *and* column 1, both weakly. So B will choose column 3, which is just the strategy we came up with by backward induction in the simple extensive-form game. And A's best response to this is to enter and pre-empt B; again what we saw above.

The case-let does mention the possibility that A might let B make the first move. The text even talks about the "luxury" of responding. But it is no luxury in this case: Without bothering about the cost information, if B makes the first move, the "game" is as shown in Figure S2.2. And a simple application of backward induction tells you: B, moving first, wants to enter, *in order to pre-empt A*. Some caution is in order about this prediction: If B enters, A's payoffs *according to the model* are -0.5 if it enters and 0 if it does not, so the conclusion (by B) that it can pre-empt A may be shaky. Still, as long as B thinks there is a reasonable probability that by entering first, it pre-empts A, then entering first is worthwhile. And A, coming to that conclusion, ought to conclude that, in this case *and based on the numbers in this model*, it should move first and pre-empt B.

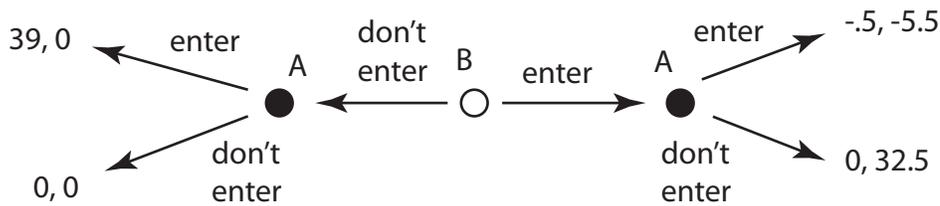


Figure S2.2. If Firm B is allowed to move first.

There are few "it always works" rules in these situations, but you're looking at a pretty general phenomenon here: If we tried to depict this situation where A and B had to make choices simultaneously—each one deciding the enter or not—we'd get the simple strategic form game in Figure S2.3.

		B's choice	
		enter	don't enter
A's choice	enter	-0.5, -0.5	39, 0
	don't enter	0, 32.5	0, 0

Figure S2.3. If the entry decision were simultaneous.

There are two Nash equilibria, one good for A, and the other good for B. In cases like this, it is generally the case that being a first-mover is advantageous—you get to choose the equilibrium outcome you prefer.