Chapter 8 Material

Solutions to the problems from Chapter 8 follow, together with a "bonus" discussion of the strange marginal-revenue function that you will discover in the last part of the Malvino Bakery problem.

8.1 This is price discrimination: US students have more inelastic demand for textbooks than do students in Great Britain.

8.2 (a) A price increase of \$0.10 for nonseniors is a 1% increase, which means it will decrease sales to nonseniors by 1.5%, or 120 units. To compensate for these 120 units fewer units sold to nonseniors, it must sell 120 units more to seniors, which on a base of 2000 units is 6%. So it must reduce price by 6% / 3 = 2%, which is \$0.20.

(b & c) Since production quantity doesn't change, total costs will not change, and we only need to worry about the change in total revenues. (This is the answer to part c.) And the change in total revenues can be computed in either of two ways:

1. Old revenues were \$100,000. New revenues are

   $$(\$9.80)(2120) + (\$10.10)(7880) = \$100{,}364;$$

   revenues, hence profits, increase by \$364.

2. Using the formula for marginal revenue $\text{MR} = P(1 + 1/\nu)$, we have a marginal revenue (at \$10) of \$6.67 for seniors and \$3.33 for nonseniors. 120 more units sold to seniors and 120 less sold to nonseniors will change revenue by

   $$\$6.67 \times 120 - \$3.33 \times 120 = \$400.$$

Either answer is acceptable.

8.3 I will solve this using Excel and the Solver add-in.

(a) Figure S8.1 shows the first sheet in the spreadsheet I created. (You should be able to replicate this.) Basic data are supplied in rows 1, 2, and 3.: The total quantity sold at a price of \$10, or 10,000 units; the overall market elasticity of -3; and the current price, \$10. The formula $\text{MR} = P(1+1/\hat{\nu})$ is used in row 6 along with the assumption that a \$10 price is profit-maximizing to compute that the implied marginal cost is \$6.67. Then data are supplied in rows 8 an 9 concerning the fraction of the buying population (by their purchase volume) that has access to the coupon and their average elasticity. These data are used in row 11 with the formula for overall elasticity from pieces to back out the implied elasticity of demand of the rest of the population.

	A	B	C
2		quantity sold	10000
3		market elasticity	-3
4		current price	$10.00
5			
6		implied MC	$6.67
7			
8		percentage getting coupon	30%
9		average elasticity this group	-6
10			
11		implied elasticity others	-1.714
12			
13		new posted price	$10.00
14		coupon value	$0.50
15			
16		sales w/o coupons	7,000.00
17		sales w/ coupons	3,900.00
18			
19		total sales	10,900.00
20			
21		implied changed in profits	$1,050.00
22		% change in profits	3.15%

Figure S8.1. Problem 8.3: The basic spreadsheet.

A posted price and a coupon value are supplied in rows 13 and 14, and sales to the two groups are computed. We begin with a posted price of \$10 and a coupon value of \$0.50. Note that the group without the coupons, since they face the original price of \$10, will continue to purchase 7000 units, although the formula for cell C16 will respond to changes in in cell C13 (the posted price), using the implied elasticity of this group (C11). As for the the 30% who access coupons, their effective price is (for these numbers) \$9.50, at which price (given their elasticity of demand) they will purchase 3900 units. The implied change in profits and the percentage change in profits are then computed, giving the answer to part a.

Figure S8.2, which is the second sheet of my spreadsheet, provides answers to parts b, c, and d. Column C of sheet 1 (Figure S8.1) is copied three times into columns D, E, and F. In column D, I ask Solver to maximize cell D21 varying D14, which provides the answer to part b. In column E, I ask Solver to maximize cell E21 by varying both E13 and E14, which provides the answer to part c. And, in column F, I change percentage of the population that gets the coupon and their average elasticity (as per part d of the problem), and ask Solver to maximize F21 by varying F13 and F14, which provides the answer to part d.

	B	C	D	E	F
2	quantity sold	10000	10000	10000	10000
3	market elasticity	-3	-3	-3	-3
4	current price	$10.00	$10.00	$10.00	$10.00
5					
6	implied MC	$6.67	$6.67	$6.67	$6.67
7					
8	percentage getting coupon	30%	30%	30%	40%
9	average elasticity this group	-6	-6	-6	-5.2
10					
11	implied elasticity others	-1.714	-1.714	-1.714	-1.533
12					
13	new posted price	$10.00	$10.00	$11.25	$11.59
14	coupon value	$0.50	$0.83	$2.08	$2.30
15					
16	sales w/o coupons	7,000.00	7,000.00	5,500.00	4,533.33
17	sales w/ coupons	3,900.00	4,500.00	4,500.00	5,466.67
18					
19	total sales	10,900.00	11,500.00	10,000.00	10,000.00
20					
21	implied change in profits	$1,050.00	$1,250.00	$3,125.00	$3,372.35
22	% change in profits	3.15%	3.75%	9.37%	10.12%

Figure S8.2. Problem 8.3, parts b, c, and d.

The numbers speak for themselves: In particular, the firm is better off with couponing 40% of the population with an elasticity of -5.2 than with couponing 30% with elasticity -6. This was not clear, a priori: A fraction closer to 50% of the market gets the coupons, which is a good thing, but their elasticity is closer to the overall figure of -3, which is a bad thing. In fact, if the numbers for the second program were that 40% of the population gets coupons, with an elasticity for that group of -5.1 (instead of -5.2), the optimized program would give us only a 9% increase in profit, slightly worse than the optimized values for the first program where 30% with an elasticity of -6 obtain coupons. I close with three comments.

1. It would be relatively easy to adapt this spreadsheet to accommodate multiple couponing programs; that is, where one segment of the population gets a high-discount coupon and a second gets a low-discount coupon.

2. The answers obtained do not depend on the original number of units sold or the price charged for them, at least insofar as the percentage change

in profit would not be sensitive to those parameters. All the important calculations in this problem can be conducted in percentage terms (What percentage of price should the coupon be? What percentage rise should be used for the posted price? What percentage change is there in profit?), which is why I included that final row.

3. The key numbers in determining the "power" of an optimized couponing scheme are the overall elasticity of demand, the percentage size of the market that will get the coupons, and the elasticity of that group. These, of course, are all "guesstimates" by product-marketing folks, and you might worry how sensitive profit changes are to mis-estimating these quantities. So in sheet 3 of the spreadsheet, shown in Figure S8.3, I do some sensitivity analysis. I fix the posted price at \$11.25 and the coupon value at \$2.08, which are (close to) the values that would be set if marketing estimated that 30% of the population with an elasticity of -6 would get the coupons, and I try those values against a variety of alternatives for the percentage getting the coupon and their elasticity.

	A	B	C	D	E	F	G	H
1								
2		quantity sold	10000	10000	10000	10000	10000	10000
3		market elasticity	-3	-3	-3	-3	-3	-3
4		current price	\$10.00	\$10.00	\$10.00	\$10.00	\$10.00	\$10.00
5								
6		implied MC	\$6.67	\$6.67	\$6.67	\$6.67	\$6.67	\$6.67
7								
8		percentage getting coupon	35%	25%	30%	30%	25%	35%
9		average elasticity this group	-6	-6	-5	-7	-5	-7
10								
11		implied elasticity others	-1.385	-2.000	-2.143	-1.286	-2.333	-0.846
12								
13		new posted price	\$11.25	\$11.25	\$11.25	\$11.25	\$11.25	\$11.25
14		coupon value	\$2.08	\$2.08	\$2.08	\$2.08	\$2.08	\$2.08
15								
16		sales w/o coupons	5,375.00	5,625.00	5,125.00	5,875.00	5,312.50	5,812.50
17		sales w/ coupons	5,243.00	3,745.00	4,245.00	4,743.00	3,537.50	5,533.50
18								
19		total sales	10,618.00	9,370.00	9,370.00	10,618.00	8,850.00	11,346.00
20								
21		implied changed in profits	\$4,427.06	\$1,822.90	\$782.90	\$5,467.06	-\$128.83	\$7,159.49
22		% change in profits	13.28%	5.47%	2.35%	16.40%	-0.39%	21.48%

Figure S8.3. Sheet 3 of Problem 8.3: Some sensitivity analysis.

Again, the numbers speak for themselves: The danger is if the firm is overly optimistic about both the fraction of the population that will get the coupons and their elasticity; cf. column G. If the firm is concerned about this, it might try to be a little less aggressive in its couponing program: How about trying a posted price of \$10.75 and a coupon value of \$1.50? In sheet 4 of the spreadsheet, shown in Figure S8.4, I try that price and coupon value for a variety of scenarios about the size of the group getting the coupon and the group's elasticity. This isn't optimal against the "best guess" of 30% and -6, but it doesn't do badly against those values, and it does provide a measure of protection against the sort of mis-estimation that is troublesome in Figure S8.3.

	A	B	C	D	E	F	G	H	I
1									
2		quantity sold	10000	10000	10000	10000	10000	10000	10000
3		market elasticity	-3	-3	-3	-3	-3	-3	-3
4		current price	$10.00	$10.00	$10.00	$10.00	$10.00	$10.00	$10.00
5									
6		implied MC	$6.67	$6.67	$6.67	$6.67	$6.67	$6.67	$6.67
7									
8		percentage getting coupon	30%	35%	25%	30%	30%	25%	35%
9		average elasticity this group	-6	-6	-6	-5	-7	-5	-7
10									
11		implied elasticity others	-1.714	-1.385	-2.000	-2.143	-1.286	-2.333	-0.846
12									
13		new posted price	$10.75	$10.75	$10.75	$10.75	$10.75	$10.75	$10.75
14		coupon value	$1.75	$1.75	$1.75	$1.75	$1.75	$1.75	$1.75
15									
16		sales w/o coupons	6,100.00	5,825.00	6,375.00	5,875.00	6,325.00	6,187.50	6,087.50
17		sales w/ coupons	4,800.00	5,600.00	4,000.00	4,500.00	5,100.00	3,750.00	5,950.00
18									
19		total sales	10,900.00	11,425.00	10,375.00	10,375.00	11,425.00	9,937.50	12,037.50
20									
21		implied changed in profits	$2,775.00	$3,518.75	$2,031.25	$1,156.25	$4,393.75	$682.29	$5,407.29
22		% change in profits	8.32%	10.56%	6.09%	3.47%	13.18%	2.05%	16.22%
23									
24		vs. % change of program with price of $11.25 and coupon $2.08	9.37%	13.28%	5.47%	2.35%	16.40%	-0.39%	21.48%

Figure S8.4. Sheet 4 of Problem 8.3: More sensitivity analysis.

8.4 (a) The overall elasticity, by formula, is

$$\frac{300}{390} \times (-1) + \frac{90}{390} \times (-5) = -1.923.$$

(b) Marginal revenue at \$2.50, by formula, is

$$\$2.50\left[1 + \frac{1}{-1.923}\right] = \$1.20.$$

This is the marginal cost.

(c) If demand by the tourists is $D_T(p) = 120(5 - p)$, elasticity of demand at \$2.50 (and quantity $120(5-2.50) = 300$) is $\$2.50/[300 \times (1/-120)] = -1$. And if demand by the locals is $D_L(p) = 180(3 - p)$, elasticity of demand at \$2.50 (where the quantity is $180(3 - 2.50) = 90$) is $\$2.50/[90 \times (1/-180)] = -5$.

If the bakery could set separate prices for the two groups, then for the tourists, whose inverse demand is $P_T(x) = 5 - x/120$ and whose marginal revenue is (therefore) $5 - x/60$, it would set MR = MC or $5 - x/60 = 1.2$ hence $x = 60 \times 3.8 = 228$, which gives a price $5 - 228/120 = \$3.10$, for a profit of $228 \times [3.1 - 1.2] = \$433.20$. And for the locals, whose inverse demand is $P_L(x) = 3 - x/180$ and whose marginal revenue is (therefore) $3 - x/90$, it would set $3 - x/90 = 1.2$, hence $x = 90 \times 1.8 = 162$, which gives a price for the locals of $3 - 162/180 = \$2.10$, for a profit of $162 \times [2.1 - 1.2] = \$145.80$.

Therefore, if it could discriminate between the two groups, it would make a daily profit of $\$145.80 + \$433.20 = \$579$, versus the \$507 in profit it makes by selling 390 loaves at \$2.50 apiece.

(d) Demand in total is the (horizontal) sum of the two demand functions. This is

$$D(p) = \begin{cases} 0, & \text{for } p > \$5, \\ 120(5-p), & \text{for } \$3 < p \le \$5, \text{ and} \\ 120(5-p) + 180(3-p), & \text{for } \$0 \le p \le \$3. \end{cases}$$

Note that there is no demand for prices above \$5, demand only from tourists for prices between \$5 and \$3, and demand from both groups for prices below \$3.

It will help to graph this demand function. And to do that, it will help to record the levels of demand at the prices \$3 and \$0. At \$3, demand is $120(5-3) = 240$, and at \$0, it is $120 \times 5 + 180 \times 3 = 1140$. Hence we have the demand function depicted in Figure S8.5.

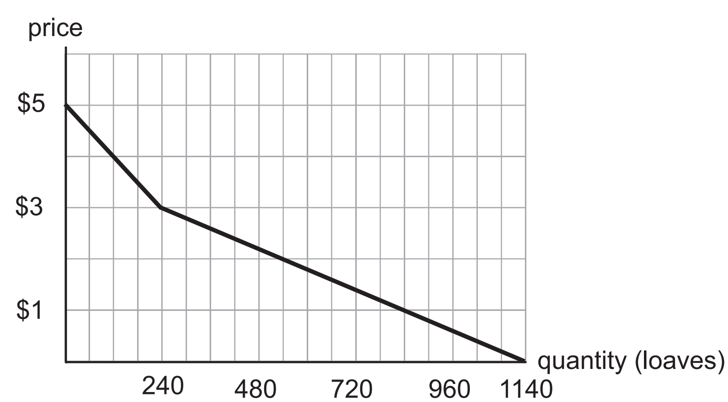

Figure S8.5. Aggregate demand for Problem 8.1. In this problem, demand comes from two different segments of the population, each of which has linear demand. So total demand is “kinked” at the price \$3 where the second group (the locals) begin to demand the product.

The next step is to invert this demand function, to find the inverse-demand function. This is why the drawing is helpful: It tells us that the three quantities 0, 240, and 1140 are crucial: Inverse demand consists of two linear functions—the inverses of my two linear demand "segments"—for the ranges of quantities $0 \leq x < 240$ and $240 \leq x \leq 1140$.

- For the range $0 \leq x < 240$, corresponding to prices in the range \$5 to \$3, demand is $120(5 - p)$. So, inverse demand in this range is $P(x) = 5 - x/120$.

- For the range $240 \leq x \leq 1140$, corresponding to prices in the range \$3 to \$0, demand is $120(5-p)+180(3-p) = 1140 - 300p$. So, inverse demand over this range is $P(x) = 1140/300 - x/300 = 3.8 - x/300$.

Summarizing, inverse demand is

$$P(x) = \begin{cases} 5 - x/120, & \text{for } 0 \leq x < 240, \text{ and} \\ 3.8 - x/300, & \text{for } 240 \leq x \leq 1140. \end{cases}$$

(If you want to add that inverse demand gives a price of \$0 for quantities greater than 1140, that's fine. Or leaving it out is fine too. Also, because demand and inverse demand are continuous at the crucial prices and quantities, you can be sloppy about which inequalities defining the ranges are weak and which are strong.)

Now total revenue is a snap:

$$\mathrm{TR}(x) = \begin{cases} 5x - x^2/120, & \text{for } 0 \leq x < 240, \text{ and} \\ 3.8x - x^2/300, & \text{for } 240 \leq x \leq 1140. \end{cases}$$

And so too, *with one caveat*, is marginal revenue: You simple take the derivative of total revenue:

$$\mathrm{MR}(x) = \begin{cases} 5 - 2x/120, & \text{for } 0 < x < 240, \text{ and} \\ 3.8 - 2x/300, & \text{for } 240 < x < 1140. \end{cases}$$

The caveat is that the total revenue function is kinked (not smooth) at the crucial quantity level $x = 240$, so it doesn't have a derivative there. Therefore, marginal revenue is not really defined for $x = 240$.

Marginal revenue is, however, very nicely defined for x strictly between 0 and 240, and for x strictly between 240 and 1140, and we can draw in the two

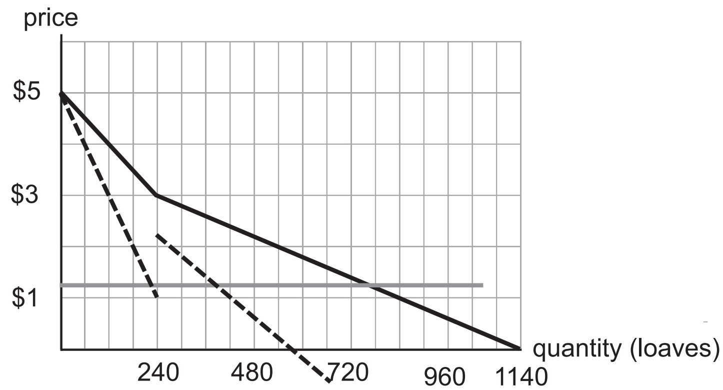

Figure S8.6. Adding marginal revenue and marginal cost to the picture in Figure 8.2. Because of the kink in the demand function (solid heavy line segments), marginal revenue (dashed line segments) is discontinuous at the quantity where the kink occurs. Indeed, marginal revenue isn't really defined at that one point. The constant marginal cost of $1.20 is also supplied, in a solid halftone line. In this case, marginal revenue jumps up at the kink in demand, and so there are two places where MR = MC, and thus two candidates for the profit-maximizing scale of production.

"segments" of marginal revenue. I've supplied the marginal revenue function, together with the constant marginal cost, in Figure S8.6, with dashed lines for marginal revenue and a halftone solid line for marginal cost.

Before equating marginal cost and marginal revenue, note that marginal revenue jumps up at the quantity 240, which is where the locals begin to buy. This may seem artificial and, in fact, it is an artifact of our model with two groups, each with linear demand functions but which different "starting-to-buy" prices. But the idea that marginal revenue could fall, then rise (if not discontinuously), and then fall again is not at all absurd; see the discussion of this phenomenon that follows the solution to this problem.

Where does marginal cost equal marginal revenue? A look at Figure S8.6 tells you that this happens in two places. We get one intersection for quantities below 240, and we can compute the quantity where this happens by setting

$$5 - \frac{2x}{120} = 1.2 \quad \text{or} \quad 3.8 = \frac{2x}{120} \quad \text{or} \quad x = \frac{120 \times 3.8}{2} = 228;$$

and a second intersection occurs above 240, which we find by setting

$$3.8 - \frac{2x}{300} = 1.2 \quad \text{or} \quad 2.6 = \frac{2x}{300} \quad \text{or} \quad x = \frac{2.6 \times 300}{2} = 390.$$

But which of these is the answer? They are both local maxima of the profit function—places where marginal revenue crosses marginal cost from above to below—but only one (except for a fantastic coincidence) will be the global max. And to find which it is, we have to give up on calculus—it has taken us as far as it can go—and evaluate profit at the two candidate solutions.[1]

For the first, 228 loaves, the corresponding price per loaf is $5 - 228/120 =$ $\$3.10$, and so profit at this point is revenue less cost, or $\$3.10 \times 228 - \$1.20 \times 228 = \$433.20$. And for the second candidate, 390 loaves, the corresponding price is $3.8 - 390/300 = \$2.50$, so profit at this point is $\$2.50 \times 390 - \$1.20 \times 390 = \$507$. The larger number of loaves gives the greater profit; the global maximizer—the profit-maximizing production level—is 390 loaves.

Please note that it doesn't always turn out that the larger production rate gives more profits. If marginal cost were \$2.00, the lower of the two intersections (which would be less than 228 loaves, because of the higher marginal cost) would be the answer.

Bottom line: If the bakery has to set a single price for both groups, it should set the price at \$2.50 (which it has been doing—the folks at the bakery are quite smart), for a total profit of \$507. But if they can discriminate between the two groups, they can push their profit up to \$579 (per week). Price discrimination for them would be quite profitable.

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Do Real-Life Marginal Revenue Functions Ever Increase?

An interesting feature of the analysis just given is that, because of the kink in the demand and inverse demand functions, marginal revenue jumps up. This, ultimately, is the source of the multiple solutions of MC = MR. But this is only a toy example. You might wonder whether, in real-life applications, you can ever get multiple solutions of MC = MR, because of a marginal revenue function that, over some range at least, is increasing.

This can happen. Which means that a firm, even if it knows the demand function it faces near the price it charges (that is, even if it knows the elasticity of demand it faces where it is) might be at a local profit-maximizing price

[1] If you remember about integrals, you can see from the picture that the second intersection point gives higher profit. Total profit is the integral of marginal profit which is marginal revenue less marginal cost. So, from $x = 0$ to the first intersection, profit is rising. From that first intersection to $x = 240$, marginal cost exceeds marginal revenue, so profit is falling. From $x = 240$ to the second intersection, profit is rising again, and then it falls. Hence the two intersections are local maxima. But, more than that, the "triangular area" bounded by the marginal-cost function and the marginal-revenue function from the first intersection to $x = 240$ is how much profit falls over that interval, while the triangular area from 240 up to the second intersection measures how much it rises. Clearly, if falls much less than it rises—the first triangle has much smaller area than the second—so profit is globally maximized at the second intersection.

and quantity, rather than a global max. Here is how.

The formula $\mathrm{MR}(x) = P(x)[1 + 1/\hat{\nu}(x)]$ says that marginal revenue depends on price and elasticity. Of course, price is (generally) a decreasing function of quantity. But elasticity can change dramatically and in different directions, as quantity increases. In particular, demand can be fairly inelastic for a range of quantities, become fairly elastic for larger quantities, then become inelastic again, at least for some goods. Where demand is inelastic, equation the formula tells us that marginal revenue is negative, so this pattern means that marginal revenue is negative, positive, and then negative. This certainly is inconsistent with downward sloping marginal revenue.

An example concerns textbooks such as this one. As I discussed with publishers pricing strategies for the first edition of this book, I was told (by many publishers, so I assume this is correct) that, if we charge a very, very high price relative to similar textbooks, most sales would be to libraries; at this price, the book would be too expensive for adoption by instructors in courses. This sort of textbook "sells" in a range of prices perhaps \$20 wide, because this is where the competition has settled. If priced in this range, the book would, the publisher and I hoped, rack up significant sales as an adopted textbook. For prices below the lowest price in this range, the publishers wecre all convinced that sales would not increase significantly as price declines; instructors would be no more likely to adopt the book for a lower price, once the price was in the "reasonable" range. Of course, a lower price means some increase in sales—students might be more likely to buy an unusued copy than a used copy, and greater nontextbook sales might be realized—but since most sales are as a textbook, the key to pricing the book was to hit a price in the "reasonable" range.

This is phrased in terms of prices, but as price and quantity are decreasing functions of one another, it implies the pattern of elasticities mentioned; inelastic demand for quantities that correspond to prices above or below the range, with a substantial piece of elastic demand within the range.

Having introduced this example, I can use it to make two other points.

1. I am not as convinced as the textbook publishers that demand remains inelastic for prices well below the standard range. In 2000 (when I first wrote these lines), the relevant range within the United States was \$80 to \$100. (As the second edition is being prepared, in 2018, I gather the range is at least 50% higher.) I concede that lowering the price *a bit* below \$80 was unlikely to increase sales much; demand is probably fairly inelastic over the range from, say, \$70 to \$80. But I wonder whether demand be-

comes very elastic again over the range \$30 to \$50, because in that range, perhaps, students are a lot more apt to keep their copies of the book, so that new sales replace the sale of used books. Unfortunately, no textbook publisher that I knew was willing to run such a drastic experiment. The point is that if I was right, this may be a case where publishers, just like Excel, were stuck at a local maximum that is not a global maximum.

And, rolling the clock forward to 2018, I am even more convinced that a very low price—I'll stick with the range \$30 to \$50—will (today) dramatically increase sales volume: The used book market is, if anything, better organized than in 2000, and in addition there is now the textbook rental market with which to contend. I bet that at a price of, say, \$35, it would no longer be worth hassling with used or rented books for many students. And, in fact, I was able to convince the publisher of another book of mine—one targeted at Ph.D. students—that this would be so; I believe the data so far support my hypothesis, although I admit that Ph.D. students are much more likely to hold on to their textbooks than are students pursuing a professional degree in management. Still, I'm discussing this with the publisher: If you paid a much lower price for a brand-new copy of the textbook than you would otherwise have expected, my argument wth the current publisher prevailed. If not, . . .

2. The profit-maximizing exercises we have been doing assume that firms know the relationship between the price they charge and the number of units they sell. This, in fact, is the basis for the observation made in Chapter 6 that it does not matter for our models whether you think of the firm choosing the profit-maximizing price or the profit-maximizing quantity. In some cases, this is untrue, even as an approximation. For a brand new textbook, publishers are uncertain how many copies will be sold, largely because the number of adoptions is highly uncertain. Does this mean that publishers cannot use the ideas of this chapter? Not at all. While quantities are very unclear, publishers believe they have a very good fix on how elasticity varies with price. For publishers of new textbooks, price and quantity are not equally good in the role of the driving variable. Given what they know—or at least what they think they know—they are quite willing and, they believe, able to think in terms of the profit-maximizing price to set, as they hold their breath about quantity.

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8.5 (a) For each price p (per unit), you must figure out how many units each individual will buy, which then tells you how many you can sell. To do this, you first should compute for each consumer the incremental value the consumer gains for each (additional or incremental) unit he/she buys. For instance, for the first consumer, the first unit she buys gives her an incremental $10 in value. the second gives her $8, the third $5, the fourth $2, and the fifth $1. So if the price is, say, $3, it makes sense for her to buy three units but no more. Because the units come as units—no fractions—this isn't quite marginal reasoning, but it is its near neighbor, incremental reasoning.

So, in Table S8.1, I give you the incremental values for each of the three consumers, and in Table S8.2, I turn these into the number of units sold at each integer level price (assuming the consumer buys when she is indifferent between buying or not). Note that whether the marginal cost of production is 2 or 4, the optimal price to set is $19.

	1st unit	2nd unit	3rd unit	4th unit	5th unit
consumer 1	10	8	5	2	1
consumer 2	20	19	5	3	2
consumer 3	30	20	10	2	0.5

Table S8.1. Incremental values of units to the three consumers in Problem 8.5.

(b) If the seller posts a per-unit price of $15 and an entry fee of $5: Suppose that consumer 2 pays the entry fee. That is a sunk cost for her, and she'll then behave as indicated in Table S8.1; she'll purchase two units. Those two units give her a gross gain in value of $39, and she must pay $30 for them, so gross of the entry fee, she gains $9. Hence she will indeed be willing to pay the entry fee of $5; she'd pay any entry fee up to $9. Consumer 1 won't buy any, so she certainly won't pay the entry fee. And Consumer 3 would buy 2 units (if she pays the entry fee), giving her a gross gain of $50, less the $30 she must pay in per-unit fees, for a gross gain of $20—she'll pay any entry fee up to $20 (if the per unit fee is $15), so she'll certainly pay $5. So at the arrangement of a per-unit price of $15 and an entry fee of $5, both consumers 2 and 3 will pay the entry fee, both will buy two units, and so the vendor makes

$10 (in entry fees) +4×$15 (four units sold at $15@) $-$ 4× cost of production.

This is a net profit of $54 if the cost is $4@, and $62 if the cost is $2@.

It is worth noting here that she could up the entry fee from $5 to $9 and still sell 2 each to consumers 2 and 3, which would increase the profit levels we just computed by $8. This is clearly better than just setting a per-unit fee.

Price charged	Number of units purchased by Consumer 1	Consumer 2	Consumer 3	Total sold	revenue	cost at \$2 @	profit at \$2 cost @	cost at \$4 @	at \$4 cost
30	0	0	1	1	\$30	\$2	\$28	\$4	\$26
29	0	0	1	1	\$29	\$2	\$27	\$4	\$25
28	0	0	1	1	\$28	\$2	\$26	\$4	\$24
27	0	0	1	1	\$27	\$2	\$25	\$4	\$23
26	0	0	1	1	\$26	\$2	\$24	\$4	\$22
25	0	0	1	1	\$25	\$2	\$23	\$4	\$21
24	0	0	1	1	\$24	\$2	\$22	\$4	\$20
23	0	0	1	1	\$23	\$2	\$21	\$4	\$19
22	0	0	1	1	\$22	\$2	\$20	\$4	\$18
21	0	0	1	1	\$21	\$2	\$19	\$4	\$17
20	0	1	2	3	\$60	\$6	\$54	\$12	\$48
19	**0**	**2**	**2**	**4**	**\$76**	**\$8**	**\$68**	**\$16**	**\$60**
18	0	2	2	4	\$72	\$8	\$64	\$16	\$56
17	0	2	2	4	\$68	\$8	\$60	\$16	\$52
16	0	2	2	4	\$64	\$8	\$56	\$16	\$48
15	0	2	2	4	\$60	\$8	\$52	\$16	\$44
14	0	2	2	4	\$56	\$8	\$48	\$16	\$40
13	0	2	2	4	\$52	\$8	\$44	\$16	\$36
12	0	2	2	4	\$48	\$8	\$40	\$16	\$32
11	0	2	2	4	\$44	\$8	\$36	\$16	\$28
10	1	2	3	6	\$60	\$12	\$48	\$24	\$36
9	1	2	3	6	\$54	\$12	\$42	\$24	\$30
8	2	2	3	7	\$56	\$14	\$42	\$28	\$28
7	2	2	3	7	\$49	\$14	\$35	\$28	\$21
6	2	2	3	7	\$42	\$14	\$28	\$28	\$14
5	3	3	3	9	\$45	\$18	\$27	\$36	\$9
4	3	4	3	10	\$40	\$20	\$20	\$40	\$0
3	3	4	3	10	\$30	\$20	\$10	\$40	-\$10
2	4	5	4	13	\$26	\$26	\$0	\$52	-\$26
1	4	5	4	13	\$13	\$26	-\$13	\$52	-\$39
0	4	5	5	14	\$0	\$28	-\$28	\$56	-\$56

Table S8.2. Finding the optimal linear price to charge.

(c) To find the best combination of entry fee and per-unit cost, you proceed as follows: For each per-unit price p, calculate how much each consumer would choose to buy *if* she pays the entry fee, then calculate how much in entry fee she would be willing to pay. This gives you (for each p) three candidates for entry fee: the level that gets all three in the door, the level that extracts all the surplus from the middle-surplus-value consumer (the smallest-surplus-value customer is ignored), and the level that gets only the highest-surplus-value consumer to pay the fee. It's ugly, but if you look at the next page, Table S8.3 provides the analysis.

To explain what you are looking at:

The first four columns are taken from Table S8.2: For each per-unit price, we have the number of units each consumer will purchase if she pays whatever entry fee is being charged.

	purchase levels			surplus			total profit if cost is $4@			total profit if cost is $2@		
per unit price	Consumer 1	Consumer 2	Consumer 3	consumer 1	consumer 2	consumer 3	3 only	2&3	all three	3 only	2&3	all three
$30	0	0	1	$0	$0	$0	$26	$26	$26	$28	$28	$28
$29	0	0	1	$0	$0	$1	$26	$25	$25	$28	$27	$27
$28	0	0	1	$0	$0	$2	$26	$24	$24	$28	$26	$26
$27	0	0	1	$0	$0	$3	$26	$23	$23	$28	$25	$25
$26	0	0	1	$0	$0	$4	$26	$22	$22	$28	$24	$24
$25	0	0	1	$0	$0	$5	$26	$21	$21	$28	$23	$23
$24	0	0	1	$0	$0	$6	$26	$20	$20	$28	$22	$22
$23	0	0	1	$0	$0	$7	$26	$19	$19	$28	$21	$21
$22	0	0	1	$0	$0	$8	$26	$18	$18	$28	$20	$20
$21	0	0	1	$0	$0	$9	$26	$17	$17	$28	$19	$19
$20	0	1	2	$0	$0	$10	$42	$48	$48	$46	$54	$54
$19	0	2	2	$0	$1	$12	$42	$62	$60	$46	$70	$68
$18	0	2	2	$0	$3	$14	$42	$62	$56	$46	$70	$64
$17	0	2	2	$0	$5	$16	$42	$62	$52	$46	$70	$60
$16	0	2	2	$0	$7	$18	$42	$62	$48	$46	$70	$56
$15	0	2	2	$0	$9	$20	$42	$62	$44	$46	$70	$52
$14	0	2	2	$0	$11	$22	$42	$62	$40	$46	$70	$48
$13	0	2	2	$0	$13	$24	$42	$62	$36	$46	$70	$44
$12	0	2	2	$0	$15	$26	$42	$62	$32	$46	$70	$40
$11	0	2	2	$0	$17	$28	$42	$62	$28	$46	$70	$36
$10	1	2	3	$0	***$19***	$30	$48	***$68***	$36	$54	$78	$48
$9	1	2	3	$1	$21	$33	$48	$67	$33	$54	$77	$45
$8	2	2	3	$2	$23	$36	$48	$66	$34	$54	$76	$48
$7	2	2	3	$4	$25	$39	$48	$65	$33	$54	$75	$47
$6	2	2	3	$6	$27	$42	$48	$64	$32	$54	$74	$46
$5	3	3	3	$8	$29	$45	$48	$64	$33	$54	$76	$51
$4	3	4	3	$11	$32	$48	$48	$64	$33	$54	$78	$53
$3	3	4	3	$14	$36	$51	$48	$65	$32	$54	$79	$52
$2	4	5	4	$17	$40	$54	$46	$62	$25	$54	$80	$51
$1	4	5	4	$21	***$45***	$58	$46	$63	$24	$54	***$81***	$50
$0	4	5	5	$25	$50	$62	$42	$60	$19	$52	$80	$47
-$1	4	5	5	$29	$55	$67	$42	$60	$17	$52	$80	$45

Table S8.3. Finding the optimal entry fee plus per unit price.

Then we compute for each consumer the gross (of entry fee) surplus value she received from being able to purchase units at that price per unit. So, looking at the row where the per unit price is 15, we see that Consumer 1 has zero surplus, consumer 2 has $9 gross surplus, and consumer 3 has $20 gross surplus (which we computed manually for part b). [Challenge: Can you figure out how I got Excel to compute these gross surplus values. The formula is remarkably simple, once you figure it out.]

It's nice that the gross surpluses are arranged so that Consumer 3 always has the most, Consumer 2 second-most, and Consumer 1 the least. Therefore, when I go to find the optimal fee to set, I can set the fee so that all three enter (by setting the fee at consumer 1's gross surplus level), so that only 2 and 3 enter (by setting it as Consumer 2's level), and so that only Consumer 3 enters (by setting it at her gross surplus level).

So the three columns under the heading "total profit if cost is $4@" tries each of these fixed fees and computes the associated profit, while the final three columns do the same thing, but with a $2@ production cost.

And now we find the optimum by inspection: If the cost is \$4 per unit to manufacture, the biggest profit figure is \$68, attained with a per-unit price of \$10 and setting the entry fee so that Consumers 2 and 3 enter; that is, at \$19. While if the cost is \$2 per unit, the highest profit is to set the per-unit price at \$1, *which is less than marginal cost*, and the entry fee at \$45.

Why would the firm ever set the per-unit price below their marginal cost (which they are doing if the marginal cost is \$2)? It has to do with the fact that, at the level of a per unit price equal to marginal cost, the marginal consumer in terms of the fixed fee, who is consumer #2, consumes *more* than the average of the higher-surplus consumers. So by lowering the per-unit price, the vendor gets more bang for the buck out of enhanced entry fees than is lost by selling per-unit at a loss. (And if that didn't make sense to you, don't worry about it. I'd bet that well over half of the community of professional economists wouldn't expect such a thing to happen.)

8.5 I'm not going to try to descibe the way this should be done; I did it in the programming language R, but you could (presumably) use other languages. Here are the answers I got: (Warning: near the optimum, the values for slightly different pricing strategies are quite close together, so the fact that I'm doing Monte Carlo simulation means I may, due to sampling error, be slightly off. That's why the answers I got are not monotonic in the correlation. It is the values that are important, not the precise prices that get me those values.)

For part a, I ran a variety of correlations, from 0.9 to -0.9, in steps of 0.15. Table S8.4 shows for each correlation the best p (one-day price) and q price for a two-day package, as well as the profit per customer that I found. Note that this confirms the intuition that bundling pays its biggest dividends when the values are negatively correlated.

Table S8.5 gives the results for the alternative model in part b.

correlation	Best p	Best q	profit per customer
0.9	49	93	30.5979
0.75	51	92	30.6182
0.6	55	94	31.1277
0.45	54	94	31.639
0.3	57	93	31.9807
0.15	56	92	32.7004
0	59	92	33.2891
-0.15	57	90	33.9573
-0.3	61	89	34.8447
-0.45	60	89	35.6194
-0.6	64	89	37.5219
-0.75	66	88	40.0504
-0.9	73	88	44.5102

Table S8.4. Best one-day price and bundled (two-day) price, and average profit per customer, for various correlations, model from part a.

correlation	Best p	Best q	profit per customer
0.9	50	88	25.6122
0.75	51	88	26.5193
0.6	53	90	27.0535
0.45	52	87	27.4851
0.3	55	87	28.0014
0.15	57	87	28.71
0	57	86	29.1859
-0.15	56	84	30.0128
-0.3	56	83	30.566
-0.45	61	83	31.563
-0.6	59	84	33.0366
-0.75	62	81	34.9607
-0.9	67	83	38.9625

Table S8.5. Best one-day price and bundled (two-day) price, and average profit per customer, for various correlations; model from part b.