

# Chapter 9 Material

---

The solution to Problem 9.1 is presented.

9.1 Before setting off into the nitty-gritty, let me list what is different between the model in the chapter and this exercise:

- In this exercise, the costs of retailing are lower for the retailer than for the manufacturer. So it isn't clear a priori whether the manufacturer will prefer to retail the goods herself or to rely on the retailer (him).
- Total cost of manufacturing is a quadratic, so the marginal cost of manufacturing is not constant. That will complicate the analysis in the third part of the exercise, just a bit.

There are three things to do. First, we "solve" for the case where the manufacturer sells directly to the public. Then we look at going via the retailer, with a simple wholesale price. And, third, we look at the case of the manufacturer charging a fixed franchise fee plus a per-unit price.

## Selling directly to the public

This is an easy exercise. Let  $P$  be the retail price and  $x$  the quantity sold, which are connected via the inverse-demand function  $P = 131 - x/100$ . Marginal revenue is  $131 - x/50$ , and marginal cost is  $11 + x/50 + 30$  (the 30 comes from the marginal cost of retailing if done by the manufacturer), so  $MC = MR$  is

$$131 - \frac{x}{50} = 41 + \frac{x}{50} \quad \text{or} \quad 90 = \frac{2x}{50} \quad \text{or} \quad x = 2250.$$

If you do the math, this gives a retail price  $P = \$108.50$  and a profit to the manufacturer of \$86,250 (which is net of both her fixed cost of manufacturing and of retailing).

## Via the retailer, with a per-unit price only

Suppose the manufacturer sets a wholesale price of  $p$ . The total cost to the retailer of buying and reselling  $x$  cars is then  $TC(x) = 1000 + 10x + px$ , so

MC = MR for the retailer is

$$131 - \frac{x}{50} = 10 + p \quad \text{or} \quad p(x) = 121 - \frac{x}{50}.$$

This is then the inverse demand function facing the manufacturer, whose marginal cost is  $11 + x/50$ , and to MC = MR for the manufacturer is

$$11 + \frac{x}{50} = 121 - \frac{x}{25} \quad \text{or} \quad 110 = \frac{3x}{50} \quad \text{or} \quad x = 1833.33.$$

This corresponds to a wholesale price of  $121 - 1833.33/50 = \$84.33$  and a retail price of  $131 - 1833.33/100 = \$112.67$ . And, if you do the accounting, this gives net profits of

\$31,611.11 for the retailer and \$85,833.33 for the manufacturer.

Comparing this regime with direct-to-the-public retailer, the manufacturer is slightly better off going directly to the public. And the price of a car is slightly lower in the direct-to-the-public regime. But because of the cost advantages of retailing by the retailer, *and despite the effects of double marginalization*, the sum of profits earned by the retailer and manufacturer in this scenario, \$117,444.44, far exceed the profit earned by the retailer in the direct-to-the-public scenario. Note, please, that this is all cost savings—fewer cars are being sold at a higher price, but it costs a good deal less to sell them.

### A franchise fee plus a per-unit price

Finally, we look at the manufacturer charging a franchise fee plus a per-unit price.

Suppose the manufacturer sets a per-unit (wholesale) price  $p$ . As long as the franchise fee is set \$1000 or more below the retailer's gross profit, the retailer will purchase

$$x(p) = 50(121 - p)$$

cars for resale. Rewrite this as  $p(x) = 121 - x/50$ , to make  $x$  the driving variable. The retailer's gross profit (gross of the franchise fee) is

$$x[P(x) - p(x) - 10] - 1000 = x \left[ 131 - \frac{x}{100} - \left( 121 - \frac{x}{50} \right) - 10 \right] - 1000 = \frac{x^2}{100} - 1000,$$

so if the manufacturer sets  $p$  so that the retailer buys  $x$  cars, the largest franchise fee she can charge is  $x^2/100 - 2000$ . Assuming she charges that largest amount, her net profit from manufacturing and selling  $x$  cars to the retailer for resale is

$$\frac{x^2}{100} - 2000 + x \left[ 121 - \frac{x}{50} \right] - \left[ 5000 + 11x + \frac{x^2}{100} \right],$$

where the three terms are the franchise fee, the variable revenue from selling the  $x$  cars, and her manufacturing cost. To maximize this, she sets

$$121 - \frac{2x}{50} - 11 = 0 \quad \text{or} \quad x = 110 \times 25 = 2750.$$

This means a wholesale price of \$66, and retail price of \$103.50, retail gross profit of \$73,625, hence a franchise fee of \$72,625, and so net manufacturer profit of

$$\$72,625 + 2750 \times \$66 - \left[ \$5000 + 11 \times 2750 + \frac{2750^2}{100} \right] = \$133,250.$$

(The retailer's net profit is, of course, \$1000.)

Let me make three points about this. By far, the most important from the perspective of economics is the third, so please take your time and understand what it is saying:

- The fact that the  $x^2/100$  term so neatly cancelled out is sheer coincidence. The first  $x^2/100$  ( $-2000$ ) is the retailer's gross profit, and that has nothing to do with the manufacturer's manufacturing cost function.
- In the chapter, we found that the manufacturer set the per-unit price in the franchise-fee regime at her marginal cost, which was a constant \$11. Because the manufacturer's marginal cost is rising, it may not be obvious, but the marginal cost of manufacturing at  $x = 2750$  is

$$11 + \frac{2 \times 2750}{100} = \$66,$$

so that rule is still true.

- But, while the rule is still true, it is less helpful in finding the optimal level of  $x$ , since the marginal cost depends on  $x$ . You can, however,

short-circuit this with the following logic. We know that the manufacturer is going to use the franchise fee to suck all the “profit” out of the retailer except for the \$1000 needed to get the retailer to sign on. So the manufacturer wants to set  $x$  to maximize the total profit earned by the two of them together. That is, she wants to make the “pie” as big as possible, as she gets the whole thing less a small \$1000 slice.

So how do we find  $x$ ? Use her cost of manufacturing plus his cost of retailing, since those will be the costs incurred. Marginal (retail) revenue is, of course,  $131 - x/50$ . The marginal cost of manufacturing is  $11 + x/50$ . And marginal cost of retailing (done by the retailer) is 10. So  $MC = MR$ , where costs are this hybrid of her manufacturing and his retailing costs, is

$$131 - \frac{x}{50} = 11 + \frac{x}{50} + 10 \quad \text{or} \quad 110 = \frac{2x}{50},$$

which gives us  $x = 2750$ . From there, calculating  $p$ ,  $P$ , and the rest of the numbers is pretty easy.

Table S9.1 following sums up the three cases.

Direct to public retailing		Going via retailer		With an "entry fee"	
Quantity	2,250	Quantity	1,833	Quantity	2,750
Mfg cost	\$85,375.00	Mfg cost	\$63,777.78	Mfg cost	\$115,875.00
retailing cost	\$72,500.00	Retailer's cost of retailing	\$19,333.33	Retailer's cost of retailing	\$28,500.00
Retail price	\$108.50	Wholesale price	\$84.33	Wholesale price	\$66.00
TR	\$244,125.00	Retail price	\$112.67	Retail price	\$103.50
TC	\$157,875.00	Retail profit	\$31,611.11	Retail gross profit	\$73,625.00
profit	\$86,250.00	Mfg profit	\$85,833.33	Mfg profit before fee	\$60,625.00
sum profits	\$86,250.00	Mfg profit after fee	\$133,250.00	Mfg profit after fee	\$133,250.00
		sum profits	\$117,444.44	sum profits	\$134,250.00

Table S9.1. Summary of the results for Problem 9.1.