

Review Problems I

The problems to follow provide you with the opportunity to review material covered in Part I of the book. Solutions to these problems are provided after all the problem statements.

I.1 (a) In the strategic form game depicted in Figure I.1(a), find two strategy profiles that are Nash equilibria. As usual, the first payoff in each cell is for the player who picks the row, the second payoff is for the column-picker. (There are more than two Nash equilibria.)

	Left	Center Left	Center Right	Right
Top	10,1	1,1	3,2	0,0
Mid	7,1	2,2	0,1	1,1
Bot	5,2	-1,-1	1,2	2,3

(a)

	Left	Center Left	Center Right	Right
Top	10,1	2,1	3,2	0,0
Mid	7,1	1,2	0,1	-1,6
Bot	5,3	-1,-1	1,2	2,2

(b)

Figure I.1. Problem I.1: Two strategic form games.

(b) In the strategic form game depicted in Figure I.1(b), what does the application of iterated dominance tell you?

(c) In the extensive form game depicted in Figure I.2, what does the application of backward induction tell you?

I.2 This problem concerns the strategic form game depicted in Figure I.3. As usual, the first payoff in each cell is for the player who picks the row, the second payoff is for the column-picker.

(a) List all the (pure-strategy) Nash equilibrium of this game.

(b) Does any row dominate any other row? If so, say which.

(c) Suppose you were picking the row, and you had the opportunity to choose a row before the person picking the column could pick a column. To be very

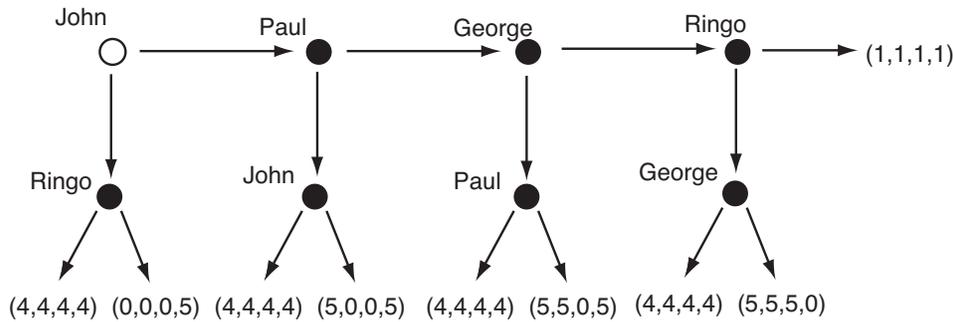


Figure I.2. Problem I.1: An extensive form game. John's payoff is listed first, Paul's second, George's third, and Ringo's last.

	Column 1	Column 2	Column 3
Row 1	3, 8	2, 3	7, 2
Row 2	4, 1	5, 2	2, 0
Row 3	0, 0	1, 3	6, 6
Row 4	0, 3	4, 5	1, 1

Figure I.3. Problem I.2.

clear, you make your choice, the column-picker sees what you chose, and then the column-picker chooses a column. Which row would you pick?

I.3 This problem concerns the game depicted in Figure I.4. Note that the payoff for the person picking the row in the Row 1–Column 1 cell is the variable X . It is not (yet) a number.

	Column 1	Column 2
Row 1	$X, 2$	1, 3
Row 2	0, 0	2, 1

Figure I.4. Problem I.3.

(a) Suppose you are the Row-picker. You are given the following option: You can play this game as a simultaneous-move game, or you can let the column-picker move first, in your sight, and then you respond. The person picking the column has had a course in game theory.

For which values of X would you prefer to play the game simultaneously? For which values of X would you prefer to have the column-picker go first? Explain the answers you give.

(b) There are no values of X for which you are strictly better off if you choose first (with the column-picker seeing your choice and then responding) than if the game is played according the rules in part (d); i.e., where you have the choice of simultaneous move or going second. Explain why this is.

I.4 Imagine a three-player game in which each player (Mae, Larry, and Curley) picks either 1, 2, or 3. This is done independently. Each player is then given a reward equal to four times the smallest number chosen, less the number chosen by the player. So if Mae chooses 2, Larry 3, and Curley 3, Mae gets $4 \times 2 - 2 = 6$, and Larry and Curley each get $4 \times 2 - 3 = 5$. This game has three Nash equilibria: What are they?

I.5 Two firms, Ace Camera and Zenith Optics, are the sole suppliers of a certain type of specialty camera used in scientific research. Each is aware of development effort that could be undertaken to improve the quality of the camera but at substantial cost. Ace, because of its larger market share, does better to undertake this development than not, regardless of whether Zenith undertakes the development. But if Zenith undertakes development as well, then Ace is worse off than if both choose to forgo development. Zenith, on the other hand, would prefer to forgo development if Ace did likewise, but if Ace develops the new product, then Zenith must as well, to remain competitive.

(a) Translating all this into the language of game theory and supposing that the two firms must choose simultaneously and independently whether to undertake development of the innovation, we get the strategic form representation of the game shown in Figure I.5.

Assuming this representation captures the situation and that both firms see the situation in this way, what prediction do you think a game theorist would make about the outcome of this competitive situation? Why?

		Zenith Optics	
		develop new product	do not develop new product
Ace Camera	develop new product	10, 2	15, 0
	do not develop new product	3, 3	12, 5

Figure I.5. Problem I.3: A strategic form game.

(b) Suppose Ace Camera could somehow commit to its action before Zenith must choose its action. (A commitment to undertake development is not

difficult. Ace could start spending money on the development process, announce that it will sell the enhanced product shortly, perhaps back up this promise with guarantees to customers, and so on. A commitment not to undertake development is more difficult, but among actions that have this character is to invest heavily in capacity for the technologically less-advanced product. For the purposes of this problem, simply assume that Ace has a commitment technology available to it.) This changes the strategic form game of part a into an extensive form game, depicted in Figure I.6. Note that, in this game, Ace chooses whether to develop the product or not, then Zenith responds, having seen Ace's choice.

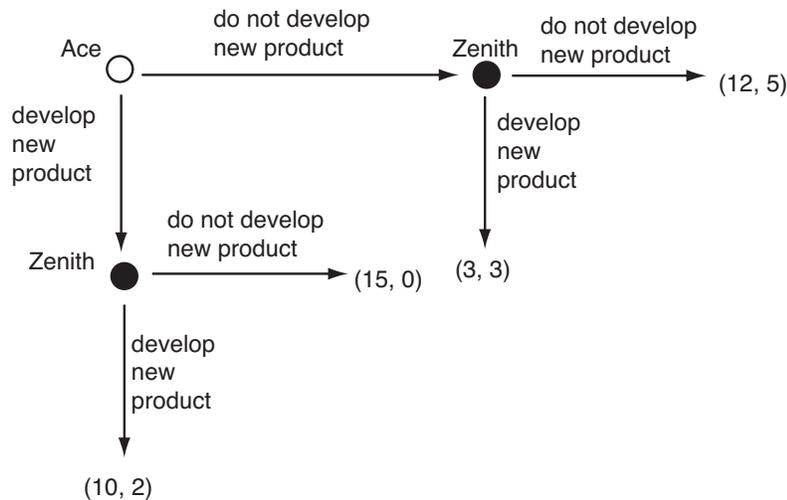


Figure I.6. Problem I.3: An extensive form game. Ace's payoffs are listed first.

Assuming this representation captures the situation and that both firms see the situation in this way, what prediction do you think a game theorist would make about the outcome of this competitive situation? Why?

I.6 Go back to the case-let from Chapter 2. What changes in the analysis, if anything, if Firm B observes not only whether Firm A entered or not, but also whether Firm A chooses to obtain the cost information. (If Firm B observes that Firm A obtained the cost information, it does not learn what Firm A learned about costs; only that they got the information. Of course, they also observe whether Firm A enters or not.)

I.7 Now consider the extensive-form depiction in Figure I.7, which is a variation on the extensive-form game in the case-let. For this extensive-form representation, what does Firm Y know when it is time to decide whether to enter or not.

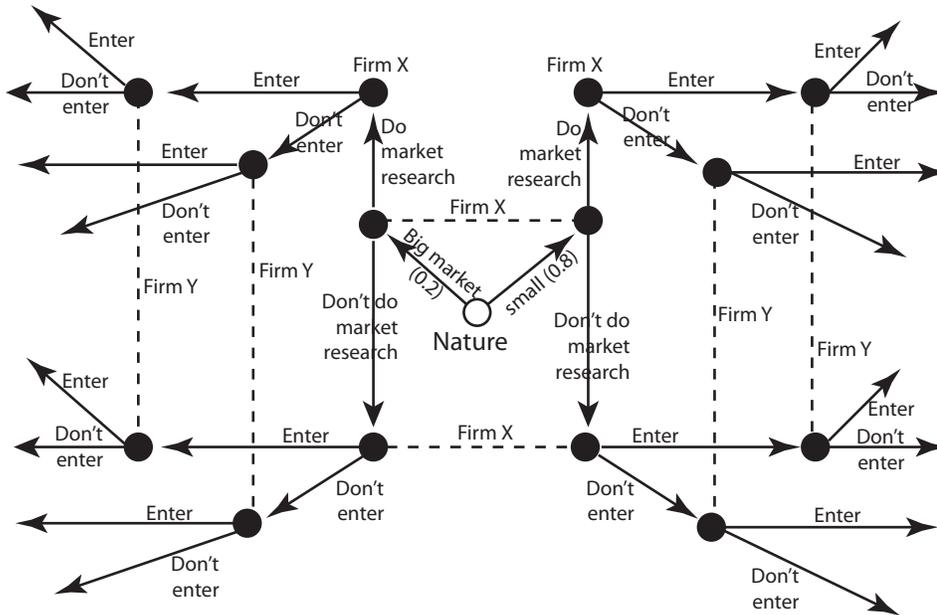


Figure 1.7. A variation on the case-let

Solution to Problem I.1

(a) Three Nash equilibria are Top–Center Right, Mid–Center Left, and Bot–Right.

(b) Top dominates Mid. Once Mid is removed, Center Right dominates Center Left and Left dominates Right. Then, Top dominates Bot (it was necessary only to remove Right on the previous step to get this far). And once Top is all that is left for the person picking the row, the person picking the column chooses Center Right.

(c) At each of the four nodes along the bottom row, the player whose turn it is would choose to go to the right. Thus along the top row: Ringo chooses to go straight right, getting (1,1,1,1); then George chooses to go straight right; then Paul chooses to go straight right; and so John chooses to begin with straight right.

Solution to Problem I.2

(a) If Row chooses row 1, Col chooses column 1, and Row chooses row 2—not a Nash equilibrium.

Row 2 → Column 2 → Row 2. Yes!

Row 3 \rightarrow Column 3 \rightarrow Row 1. No.

Row 4 \rightarrow Column 2 \rightarrow Row 2. No.

So the only Nash equilibrium is Row 2 and Column 2.

(b) Row 1 strictly dominates Row 3, and Row 2 strictly dominates Row 4.

(c) If you pick row 1, Col will choose col 1, giving you 3.

If you pick row 2, Col will choose column 2, giving you 5.

If you pick row 3, Col will choose column 3, giving you 6.

If you pick row 4, Col will choose column 2, giving you 4.

So you are best off picking Row 3.

Solution to Problem I.3

(a) If you play this game as a simultaneous-move game, then Col will choose column 2, since column 2 dominates column 1. You, therefore, would want to choose row 2, which gives you 2.

If you let Col go first, and he picks column 1, you will choose row 1 if $X > 0$ which will give him 2 and row 2 if $X < 0$, which will give him 0. If he picks column 2, you will choose Row 2, which gives him 1. So he will pick column 1 if $X > 0$ and column 2 if $X < 0$.

What about if $X = 0$? It's unclear what he'll do. If he thinks you'll pick row 1 in response, he'll pick column 1. If not, he'll pick column 2.

So if you let Col go first, and if $X < 0$, you will get 2. If $X > 0$, you will get X . If $X > 2$, you prefer this to simultaneous move. If X is between 0 and 2, you prefer simultaneous move. If $X < 0$, you don't care.

(At the two knife-edge cases: If $X = 2$, you will get 2, so you don't care. If $X = 0$, he might pick column 1, which you don't like, so you prefer simultaneous-move which ensures he'll pick column 2).

So, to reiterate:

If $X < 0$, you don't care, since you'll get 2 in any case.

If $X = 0$, you prefer simultaneous move, just to be sure that you get 2.

If $0 < X < 2$, you prefer simultaneous move to get 2.

If $X = 2$, you don't care.

If $X > 2$, you prefer that he move first, because he will choose column 1 and you will get X .

(b) If you go first and you pick row 1, your opponent will pick column 2. If you pick row 2, he will pick column 2. (Because column 2 dominates column 1, this is obvious.) So you are better off beginning by picking row 2, which gets you a payoff of 2. This is never any better than what you get in the game as played in part a.

Solution to Problem I.4

The three Nash equilibria are these: All three choose 1, netting payoffs of 3 apiece; all choose 2, netting payoffs of 6 apiece; and all choose 3, netting payoffs of 9 apiece. Note why these are Nash equilibria: Any single player, by increasing the number chosen does not affect the minimum, hence decreases her own payoff; by decreasing the number chosen, she decreases the minimum and thus decreases her own payoff. And there are no other Nash equilibria: If the three do not name the same number, any player who chooses a number other than the lowest among the three does better to decrease her number to equal the lowest.

Solution to Problem I.5

(a) For Ace, developing the new product dominates not doing it, so Ace certainly develops the new product. Zenith, anticipating this, chooses to develop the new product itself, giving payoffs of 10 to Ace and 2 to Zenith. The logic of iterated dominance applies here.

(b) In this case we use backward induction. If Ace develops the new product, Zenith will respond by developing the new product, with payoffs 10 and 2, respectively. But, if Ace commits not to develop, then Zenith is better off if it does not develop, giving payoffs of 12 and 5. So Ace chooses not to develop.

Comparing parts a and b, this is a case in which one firm would like to be able to commit to a particular action, so as to guide its rival into a corresponding response that makes *both* of them better off. Ace, if it cannot affect Zenith's action, sees development as the better (dominant) strategy. Zenith, realizing that Ace sees no way to influence its (Zenith's) actions and so will develop the new product, must develop the new product itself. But if Ace could commit in advance, it chooses the strategy that is (otherwise) dominated, because this elicits from Zenith a response (no development) that makes each side better off.

Solution to Problem I.6

The extensive form of the game changes to what is depicted in Figure I.7: The choice nodes Firm B has on the left-hand side of the tree are "disconnected" informationally from the two nodes in the center and right. (Top and bottom are still disconnected.)

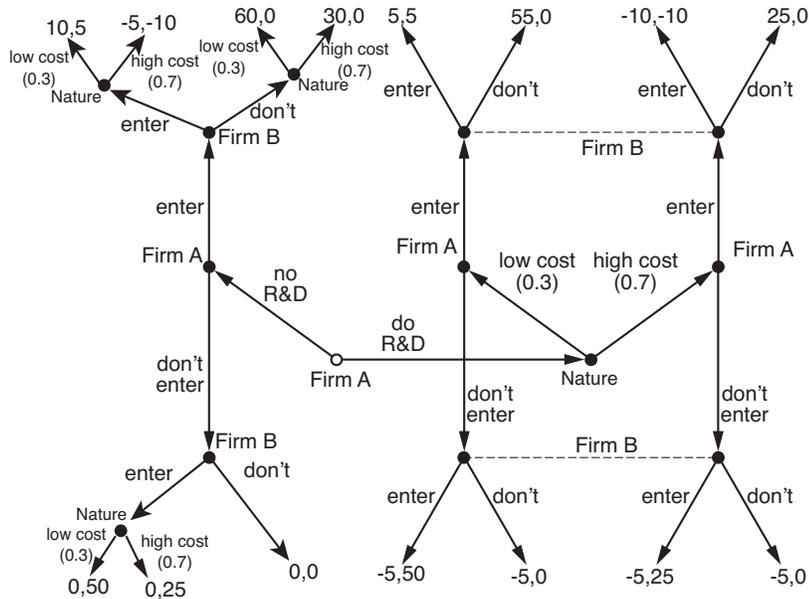


Figure I.7. Problem I.6: The revised extensive-form game.

This means that Firm B must make the enter / don't enter decision in four different places (information sets), and so Firm B has 16 possible strategies.

But we can prune some of this away, on the left-hand side of Figure I.7. If Firm A does not get the cost information, firm B sees this, and the game becomes one of complete and perfect information, which can (therefore) be solved by backward induction: If Firm A chooses to enter, Firm B will not, netting payoff 0, and Firm A will have an expected value of $(0.3)(60) + (0.7)(30) = 39$. And if firm A doesn't enter, firm B will enter (expected value $(0.3)(50) + (0.7)(25) > 0$), so firm A will net 0. Hence, once firm A chooses not to get the cost information, firm A will choose to enter, and the net payoffs will be 39 for firm A and 0 for firm B. The game in Figure I.7 reduces to the game in Figure I.8, for which Firm B only has four strategies.

Firm A, in this reduced tree, has five strategies: Don't get the information and enter, Get the info and enter regardless of what is learned, Get the info and don't enter regardless of what is learned, Get the info and enter if (and only if) costs are low, Get the info and enter if (and only if) costs are high.

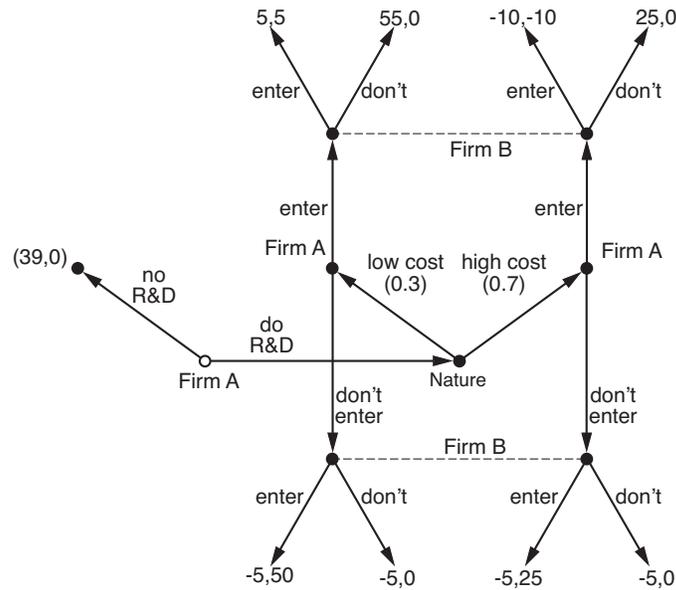


Figure 1.8. Problem 1.6: The revised and reduced extensive-form game.

For precisely the reasons given in the original solution, we can dismiss the second and third of these: Why pay for information you won't use? Now, it isn't quite so simple here: Perhaps by getting the information, Firm A can convince Firm B to stay out. But we already know that Firm B will stay out if Firm A doesn't get the information (we learned that when we reduced the left-hand side of the tree), so getting the information just to forestall Firm B is unnecessary.

And we can eliminate the strategy of getting the information and entering if (and only if) costs are high, for the same reasons as in the original solution.

Finally, we can reject the strategy of getting the information and entering only when costs are low. If Firm A does this, Firm B will know that costs are low if they see firm A getting the information and entering. And, if Firm A's actions show them that costs are low, they want to enter whether firm A enters or not. Firm A does not want firm B to enter under any circumstances, so this strategy is a loser for Firm A. (If that argument was too quick for you, construct the strategic-form game and you'll see it.)

So, in terms of "the answer," the change makes no difference: Firm A should not get the information and should enter, and Firm B should respond with no entry.

That said, I would contend (going beyond what a game-theoretic analysis tells us) that Firm A is *better off* in this game if Firm B sees whether Firm A gets the information. Suppose Firm B doesn't see whether Firm A gets the

information *and* isn't so good at game theory, simply supposing that firm A is going to get the information and enter if (and only if) costs are low. Then, seeing Firm A enter, Firm B "infers" that costs are low and chooses to enter. That is, if Firm B *knows* that Firm A did not get the cost information, and if they see Firm A enter, they know there is no information in Firm A's entry decision about costs, and if there is no information, then they definitely want to stay out, which is what Firm A wants them to do. If they don't see whether Firm A gets the cost information, they might make an incorrect inference about costs from Firm A's entry, an incorrect inference that Firm A *does not* want them to make. (The argument in this last paragraph is not something you should expect to have seen, based on what is in the chapters.)

Solution to Problem I.7

Firm Y has four information sets. Look at the two nodes in the left-most of these four. The sequence of actions leading to the top node is: It is a big market, Firm X does market research, Firm X enters. And the bottom node in this information set follows: It is a big market, Firm X doesn't do market research, Firm X enters. So at this information set, Firm Y *knows* that the market is big and that Firm X entered; the only thing it doesn't know is whether Firm X did market research or not.

If you work through the other three information sets for Firm Y, the pattern is the same: Firm Y knows the market size and knows whether Firm X entered or not; Firm Y doesn't know whether Firm X did market research. So that's what this extensive form represents: Firm Y knows market size and whether Firm X entered, but not whether X chose to do market research. (Challenge: For this representation, what is the corresponding strategic form? What does analysis of the situation suggest?)