

Review Problems III

The problems to follow provide you with the opportunity to review material covered in Part III of the book. Solutions to these problems are provided after all the problem statements.

III.1 A consumer must choose a bundle consisting of bread, cheese, and salami, the prices of which are $p_b = \$2$, $p_c = \$10$, and $p_s = \$20$. This consumer has \$20 to spend. His utility function is

$$u(b, c, s) = 6 \ln(b + 1) + 2 \ln(c + 2) + \ln(s + 3).$$

What does this consumer choose?

III.2 A consumer must choose a bundle consisting of bread, cheese, salami, and fudge, the prices of which are $p_b = \$2$, $p_c = \$10$, $p_s = \$20$, and $p_f = \$10$. This consumer has \$500 in her pocket, but she values money left over, so she may not spend the whole \$500 on this meal. Her utility function is

$$u(b, c, s) = 60 \ln(b + 1) + 30 \ln(c + 2) + 10 \ln(s + 3) + 0.2f - 0.2f^2 + m.$$

What does this consumer choose?

III.3 Part a of this problem involves finding the demand functions of individual consumers who have money-left-over utility functions, summing those functions to find overall demand, inverting the demand function to get inverse demand, and then solving $MC = MR$ to find the profit-maximizing price for the firm. This is a lot of separate steps, but if you take them one at a time, you should be able to get through this.

Part b, on the other hand, involves 1st-degree price discrimination in a B2C context. Until (if and when) you do Problem 10.14 from the text, this is probably not within reach.

a. A firm sells its product to 5000 consumers. Remarkably, each of these consumers has a utility function of the form

$$u(x, m) = kx - x^2 + m$$

where x is the amount of the good consumed, m is money left over, and k is a constant that is different for different consumers. Specifically, 1000 consumers have $k = 4$, 1000 have $k = 5$, and the other 3000 have $k = 6$. Each consumer has at least \$100 in his or her pocket. The marginal cost of producing this good is \$1.

(a) If the firm sets a price per unit p and lets consumers decide how much to purchase, what price p maximizes the firm's profit?

(b) If the firm could engage in first-degree price discrimination, what would it do, and how much profit would it make?

III.4 A firm has production function $f(k, l, m) = 16k^{1/12}l^{1/6}m^{1/4}$, for inputs k (capital equipment), l (labor), and m (material). The prices of the three inputs are, respectively, $r_k = 27$, $r_l = 2$, and $r_m = 9$. What is the total-cost function for this firm? If the inverse-demand function facing this firm is $P(x) = 600 - 2x$, what are its profit-maximizing price and quantity?

III.5 Figure III.1 shows the 100-unit isoquant of a firm that produces fasdip out of material and labor. The production process of this firm has constant returns to scale.

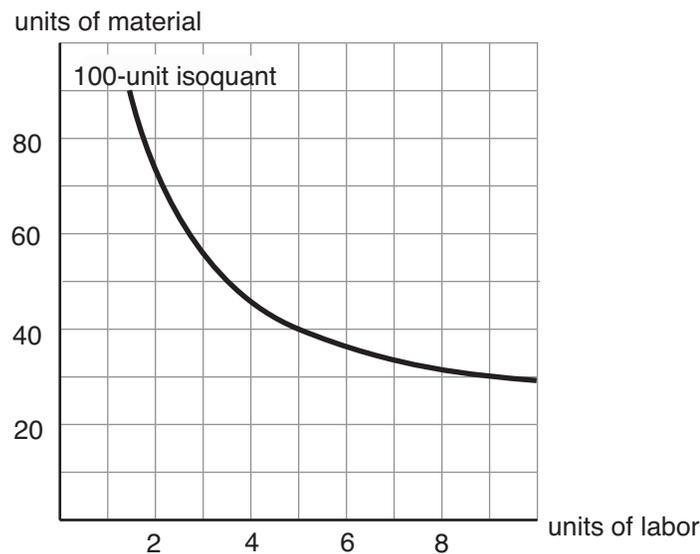


Figure III.1 Problem III.5: The 100-unit isoquant for the manufacture of fasdip

(a) If the price of materials is \$2.50 per unit and the price of labor is \$10 per unit, approximately what is the (least-cost) total cost of producing 100 units of fasdip?

(b) Suppose this firm faces the inverse demand curve

$$P(x) = 5 - \frac{x}{200}.$$

(Input prices are as in part a of the problem.) What price should it charge, and what quantity should it sell, to maximize its profit?

(c) Suppose that, in the short run, the firm in question cannot change the amount of labor it hires, although it can change the amount of material. Beginning from the profit-maximizing position you calculated in part b, find the short-run total cost of producing half the amount produced in part b. (This is difficult.)

III.6 The very strange looking isoquant in Figure III.2 goes with the following story. The firm in question has a choice of two different fixed-coefficient, constant-returns-to-scale production technologies. In the first, each unit of output requires 2 units of labor and 3.5 units of material, while in the second, each unit of output requires 3 units of labor and 2 units of material. This technology has constant returns to scale (so the one isoquant describes the technology completely). The firm must use only one of the two technologies at any time; it cannot produce some fraction of its output with one technology and the rest with the second technology. (Something worth thinking about, if you get the problem: If the firm can shift between the two technologies at negligible cost, say, using one technology one month and the other the next, does it matter that the firm is unable to use both technologies at the same time?)

(a) Suppose the price of labor is $r_l = \$1$ and the price of material is $r_m = \$1$. The firm wishes to produce 20 units of output. How many units of labor and how many units of material would the firm employ? What would be the total cost of the 20 units? (Throughout this problem, assume that this firm makes profit-maximizing choices.)

(b) Suppose that, under the same conditions as in part a, the firm wishes to produce 40 units of output. How many units of labor and material would the firm employ? What will be the total cost of the 40 units?

(c) Suppose the firm faces the inverse demand curve $P(x) = 25 - x/100$. What would the firm do (in terms of quantity produced and price charged)?

(d) Suppose the firm reaches the position you found in part c. Suddenly the price of labor doubles to \$2 per unit. In the short run, the firm can freely change the amount of labor and material it employs, but it cannot change

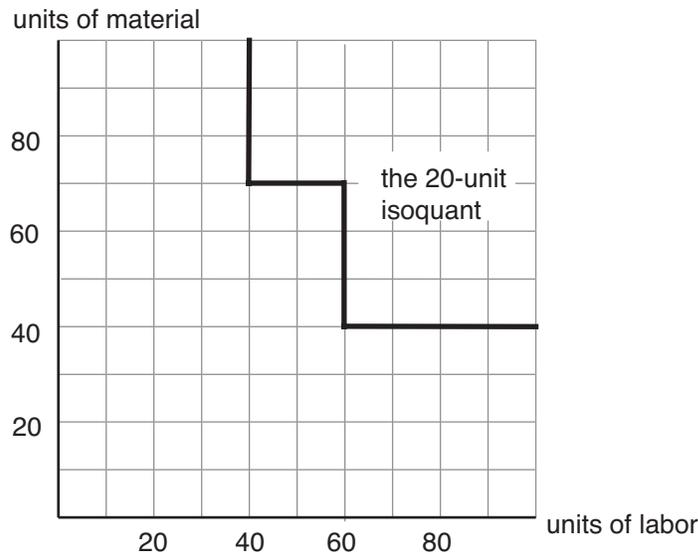


Figure III.2 Problem III.6: An isoquant.

the technology it uses; that is, if it previously used inputs in the fixed ratio of 2 labor to 3.5 capital (per unit of output), it must stay with this ratio, and if it used the fixed ratio of 3 labor to 2 material (per unit of output), it must stay with this. What does the firm do in the short run in response to the rise in the cost of labor?

(e) In the longer run, the firm is able to change technologies if it wishes to. What is its long-run response to the new price of labor?

III.7 The manufacture of trewqs requires both labor and raw material. Specifically, each trewq requires 5 hours of labor time, at a cost of \$10 per labor hour. (Therefore, x trewqs require $5x$ labor hours and has a labor total cost of $50x$.) The material requirements of trewqs is more complex. To produce x trewqs per day requires $2x + x^2/10$ units of material, at a cost of \$2 per unit. No substitution is possible between labor and materials. There are no fixed costs.

(a) What is the (long-run) total cost function for trewqs?

(b) Suppose that inverse demand function for trewqs is given $P(x) = 198 - 0.4x$. How many trewqs does the manufacturer sell to maximize profit, and what price do they sell for?

(c) The government imposes a tax of \$4 per trewq, collected directly from the manufacturer. (That is, for each trewq produced, the firm must pay the government \$4.) What effect does this have on the total cost of trewq

production? What effect does this have on the marginal cost?

(d) In the short run, although this firm can freely change the number of units of raw material that it uses (at the market price of \$2 per unit), it cannot easily change the amount of labor time it employs: It cannot, in the short run, discharge anyone it has been employing; and it can add labor hours only by paying a premium overtime wage of \$15 per hour. What is the impact of the tax of part c on the price and quantity of trewqs supplied by the manufacturer in the short run?

(e) What is the impact of the tax on the price and quantity of trewqs supplied by the manufacturer in the long run?

For the remainder of this problem, forget about the tax: You are back to the situation described in part b. Suddenly it becomes feasible to produce trewqs by a second technology, which uses inputs in fixed proportions and has constant returns to scale. In this technology, each trewq requires 4 units of labor and 6 units of material. However to use this technology, a third input, a machine known as a *bliffilator*, is required. To make one trewq requires (in addition to the 4 units of labor and 6 units of material) 6 minutes of bliffilator time. Bliffilator time can be leased by the firm, at a cost of \$140 per hour. (Bliffilator time can be rented in any units you desire, including fractions of an hour.)

(f) What is the total cost of making 120 trewqs entirely by this new method?

(g) Suppose the firm converted its production *entirely* to this new method. What quantity would it produce and what price would it charge? What would be its profit?

(h) Suppose the firm could use both technologies simultaneously. That is, it can make some of its trewqs by the first method and others by the second. (Let me be very precise here: Suppose the firm makes x_1 trewqs by the first method and x_2 by the new, second method. Then, it would require $5x_1 + 4x_2$ units of labor, $2x_1 + x_1^2/10 + 6x_2$ units of material, and $0.1x_2$ hours of bliffilator time.) To maximize its profit, what quantity would it produce and what price would it charge? What would be its profit? Explain your answer.

III.8 For a final time, consider the firm with production function $f(l, m) = l^{1/6}m^{1/3}$ for inputs l (labor) and m (material), facing inverse demand $P(x) = 160 - 2x$, with the price of l set at \$4 and the price of m set at \$1. This firm, you will recall, maximizes profit by setting $x = 16$, $l = 64$, and $m = 512$. In the short run, this firm can vary m freely but is stuck at the status-quo level

of $l = 64$. In the long run, it can change both l and m freely.

The price of m suddenly rises to \$2. How does the firm react in the short run (assuming it acts to maximize its profit)? If the price of m stays at \$2, how will the firm react (to maximize profit) in the long run?

III.9 (This problem requires that you have solved or gone through the solution of Problem 11.10.) A firm has two cost-independent facilities at which it can make a particular item. If it makes x_1 units at the first facility, its total cost there is $5x_1 + x_1^2/2000$, which means a marginal cost of $5 + x_1/1000$. If it makes x_2 units at the second facility, its total cost there is $8x_2 + x_2^2/4000$, which means a marginal cost of $8 + x_2/2000$. In addition, the firm can purchase this item from a foreign vendor, who is willing to sell the domestic firm as much as it (the domestic firm) wishes at a price of \$15 per unit.

(a) Since marginal costs are rising for this firm at both facilities, there is a limit to how many it will make domestically, before it moves to purchasing units from the foreign vendor. What is that limit?

(b) Suppose the firm wished to produce or procure 15,000 units. What would be the cost-minimizing way to do this?

(c) Suppose this firm faces inverse demand function $P(x) = 24 - x/6000$. What should the firm do to maximize its profit?

Solution to Problem III.1

Begin by assuming that the consumer consumes a strictly positive amount of each good. Then the equal-bangs-for-the-buck equation is

$$\frac{6}{2(b+1)} = \frac{2}{10(c+2)} = \frac{1}{20(s+3)}$$

which is

$$\frac{b+1}{3} = 5(c+2) = 20(s+3).$$

The budget equation is

$$2b + 10c + 20s = 20.$$

Now $10c = 2(b + 1)/3 - 20$, and $20s = (b + 1)/3 - 60$, so the budget equation can be rewritten

$$2b + \frac{2(b + 2)}{3} - 20 + \frac{b + 1}{3} - 60 = 20,$$

or

$$2b + b + 5/3 = 100,$$

or $3b = 98.333$, or $b = 32.778$. This gives a negative value for s , so the solution cannot have all three commodities strictly positive.

Perhaps, therefore, the solution has $s = 0$ and both b and c strictly positive. The equal-bangs-for-the-buck equation in bread and cheese is $(b + 1)/3 = 5(c + 2)$, and the budget equation is $2b + 10c = 20$, which becomes

$$2b + \frac{2(b + 1)}{3} - 20 = 20, \quad \text{or} \quad \frac{8b}{3} = \frac{119}{3},$$

or $b = 119/88$. Therefore, $10c = 2(b + 1)/3 - 20 = 2(119/88 + 1) - 20 = -15.29545 \dots < 0$. So this cannot be the solution.

We could try $c = 0$ and b and s strictly positive, but instead I try b strictly positive and both c and s equal to 0. Since all \$20 is spent on bread, $b = 10$. At this level of b , the bang for the buck in bread is $\frac{3}{22}$. And the bang for the buck in cheese at $c = 0$ is $\frac{2}{20} = 0.1$, while the bang for the buck in salami at $s = 0$ is $\frac{1}{60}$. So $b = 10$, $c = 0$, and $s = 0$ works: The bangs for the buck of the goods consumed in strictly positive amounts are equal—trivially so, because there is only one such good—and this bang for the buck exceeds that of the other goods, at 0 levels of consumption. That is the answer.

Solution to Problem III.2

This is a $\dots + m$ money-left-over utility function, so we begin hypothesizing that the consumer has some money left over at the end of her purchases. The bang for the buck for money left over is 1, so equating this to the bang for the buck for bread, cheese, salami, and fudge gives

$$\frac{60}{2(b + 1)} = 1 \text{ or } b = 29, \quad \frac{30}{10(c + 2)} = 1 \text{ or } c = 1,$$

$$\frac{10}{20(s+3)} = 1 \text{ or } s = \frac{-5}{2}, \quad \text{and} \quad 0.2 - 0.4f = 1 \text{ or } f = -2.$$

The last two violate the nonnegativity constraints and lead us to notice that, at $s = f = 0$, the bangs for the buck of those two commodities are already less than 1. Hence, the answer, subject to checking on whether the consumer has money left over, is $b = 29$, $c = 1$, $s = f = 0$. And, sure enough, at these quantities, the consumer's expenditure is \$68, a good deal less than the \$500 she possesses. This is indeed the answer.

Solution to Problem III.3

This is a long problem, but it contains almost everything we have done so far.

(a) To answer part a, we need to work out the total demand function facing the firm. We first find demand, consumer by consumer.

Group 1 consumers have the utility function $u(x, m) = 4x - x^2 + m$. Inverse demand for a consumer from this group is then $P(x) = 4 - 2x$, and demand is $d_1(p) = 2 - p/2$ (for prices below 4, of course).

Group 2 consumers have the utility function $u(x, m) = 5x - x^2 + m$. Inverse demand for a consumer from this group is then $P(x) = 5 - 2x$, and demand is $d_2(p) = 2.5 - p/2$ (for prices below 5).

Group 3 consumers have the utility function $u(x, m) = 6x - x^2 + m$. Inverse demand for a consumer from this group is then $P(x) = 6 - 2x$, and demand is $d_3(p) = 3 - p/2$ (for prices below 6).

(We have to check that the consumers spend no more than \$100 at any price. Let me remind you how this is done for group 3 consumers. Expenditure by a member of this group for prices from 0 to 3 is $3p - p^2/2$, so expenditure is maximized where the derivative of this function is 0, or $p = 3$. At that point, the quantity purchased is 1.5, for a total expenditure of \$4.50, which is a good deal less than \$100.)

Next we write out total demand:

$$D(p) = \begin{cases} 0, & \text{for } p > 6, \\ 3000(3 - p/2), & \text{for } 6 \geq p > 5, \\ 3000(3 - p/2) + 1000(2.5 - p/2), & \text{for } 5 \geq p > 4, \\ 3000(3 - p/2) + 1000(2.5 - p/2) + 1000(2 - p/2), & \text{for } p \leq 4. \end{cases}$$

From this we can find total inverse demand. First we find the quantities that correspond to the prices of \$5 and \$4: At $p = \$5$, total demand is 1500, while at $p = \$4$, total demand is $3000 + 500 = 3500$. Therefore,

- For quantities from 0 to 1500, inverse demand is the inverse of $x = 9000 - 1500p$, which is $P(x) = 6 - x/1500$. Note that marginal revenue over this range of quantities is $MR(x) = 6 - x/750$. Equate this to marginal costs of 1, and you get $6 - x/750 = 1$ or $x = 750 \times 5 = 3750$, which is much more than 1500. Marginal cost does not intersect marginal revenue for this range of quantities.
- For quantities from 1500 to 3500, inverse demand is the inverse of $x = 3000(3 - p/2) + 1000(2.5 - p/2) = 11,500 - 2000p$, which is $P(x) = 5.75 - x/2000$. Marginal revenue over this range of quantities is $MR(x) = 5.75 - x/1000$. Set this equal to marginal cost, and you get $5.75 - x/1000 = 1$ or $x = 4750$, which is outside the range, so that there is no intersection of marginal cost and marginal revenue for this range of quantities either.
- For quantities above 3500, inverse demand is the inverse of $x = 3000(3 - p/2) + 1000(2.5 - p/2) + 1000(2 - p/2) = 13,500 - 2500p$, which is $P(x) = 5.4 - x/2500$. Marginal revenue in this range is $MR(x) = 5.4 - x/1250$. Equate this to marginal cost, and you get $x = 1250 \times 4.4 = 5500$. Since this is the only place that marginal cost equals marginal revenue, this must be the profit-maximizing quantity. This corresponds to a price of $5.4 - 5500/2500 = \$3.20$ and a profit of $(3.20 - 1)(5500) = \$12,100$.

Figure III.3 provides a graphical depiction of this analysis.

(b) You can do this part of the problem with take-it-or-leave-it offers, or with fixed-fee plus per-unit price offers. I use the latter.

First, we work out how much to sell to each consumer. For a consumer with $u(x, m) = kx - x^2 + m$, set marginal utility equal to marginal cost: $k - 2x = 1$, or $x = (k - 1)/2$. So, for group 1 ($k = 4$), $x = 1.5$. For group 2, $x = 2$. For group 3, $x = 2.5$.

Next we compute the gross gain in utility for each sort of consumer. If a consumer with utility function $u(x, m) = kx - x^2 + m$ consumes $(k - 1)/2$ units of the x good, the gross gain in utility is

$$\frac{k(k-1)}{2} - \frac{(k-1)^2}{4} = \frac{2k^2 - 2k - k^2 + 2k - 1}{4} = \frac{k^2 - 1}{4}.$$

We set the per-unit price at marginal cost, or $p = 1$, for every consumer. And we set the fixed fee at the gross gain in utility less what the consumer must

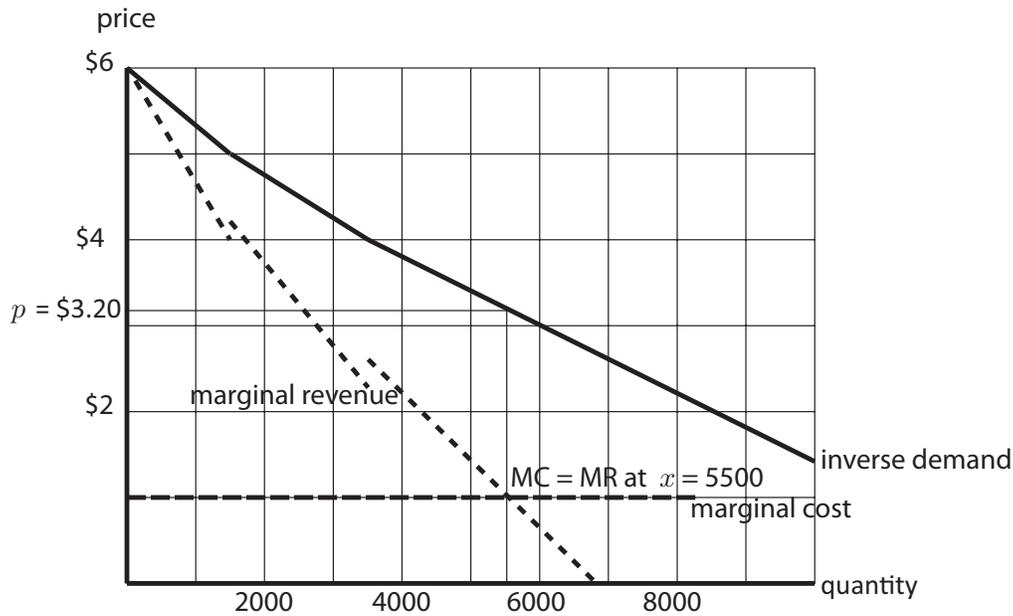


Figure III.3. Problem III.3(a).

pay for her goods:

$$\frac{k^2 - 1}{4} - \frac{k - 1}{2} = \frac{k^2 - 2k + 1}{4}.$$

The firm's profit comes entirely from the fixed fees, so

- From group 1, where $k = 4$, the firm makes $\frac{9}{4}$ in profit for each of the 1000 members of this group, or \$2250.
- From group 2, where $k = 5$, the firm makes 4 in profit for each of the 1000 members of this group, or \$4000.
- From group 3, where $k = 6$, the firm makes $\frac{25}{4}$ in profit for each of the 3000 members of the group, or \$18750.

The total profit is \$25,000. (First-degree price discrimination is really powerful.)

Solution to Problem III.4

In the usual fashion, for this Cobb-Douglas production function, the cost-minimizing ratios of k to l to m (at the given prices) are given by

$$27 \times 12 \times k = 2 \times 6 \times l = 9 \times 4 \times m \quad \text{or} \quad 81k = 3l = 9m.$$

Since $x = k^{1/12}l^{1/6}m^{1/4}$, we can replace l with $27k$ and we can replace m with $9k$ and get $x = k^{1/12}(27k)^{1/6}(9k)^{1/4} = 27^{1/6}9^{1/4}k^{1/2} = 3k^{1/2}$, and so, to produce x , the cost-minimizing production plan involves

$$k = \frac{x^2}{9}, l = 27k = 3x^2, \text{ and } m = 9k = x^2,$$

for a total cost of

$$\$27 \times \frac{x^2}{9} + \$2 \times 3x^2 + \$9 \times x^2 = 18x^2.$$

Marginal revenue is $600 - 4x$, so $MR = MC$ is

$$600 - 4x = 36x \quad \text{or} \quad 600 = 40x \quad \text{or} \quad x = 15,$$

hence $P(x) = 600 - 30 = 570$, $k = x^2/9 = 25$, $l = 3x^2 = 675$, and $m = x^2 = 225$.

Solution to Problem III.5

(a) Figure III.3 shows the \$100-isocost line, which is moved parallel until it hits the 100-unit isoquant. It does so at the point where labor usage is 5 and material usage is 40, a bill of materials that costs $\$10 \times 5 + \$2.50 \times 40 = \$150$. So the total cost of making 100 units of fasdip is \$150.

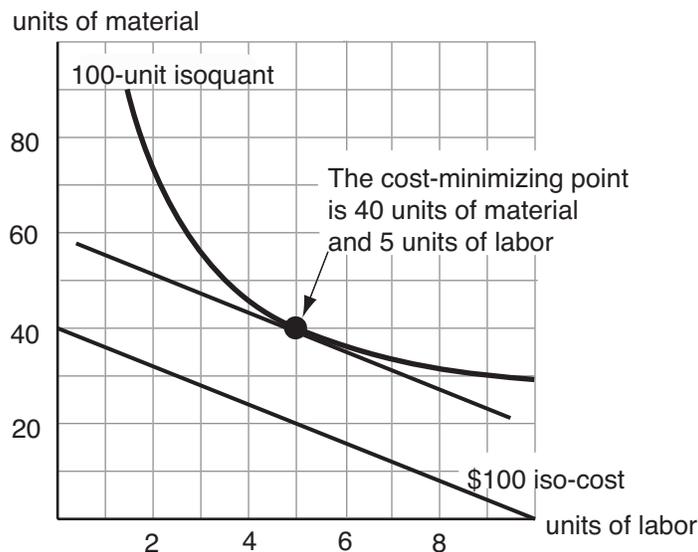


Figure III.3 Problem III.5: Finding the least-cost way to make 100 units of fasdip.

(b) Since this technology has constant returns to scale, average cost = marginal cost is constant, and from part a, we know this constant average cost = marginal cost is \$1.50. Marginal revenue (from inverse demand) is $MR(x) = 5 - x/100$, so marginal cost equals marginal revenue where $5 - x/100 = 1.5$ or $x = 350$, at which point the price is \$3.25.

(c) By constant returns to scale, $x = 350$ means $3.5 \times 5 = 17.5$ units of labor input. Hence, we want to find the level of material usage m so that the point (17.5 labor, m material) lies on the 175-unit isoquant. By constant returns to scale, this is the same m that (17.5/1.75 labor, $m/1.75$ material) lies on the $175/1.75 = 100$ -unit isoquant (we scale by 1.75 to get to the one isoquant we have). But, for 10 units of labor, we need approximately 29 units of material to be on the 100-unit isoquant, so $m/1.75 = 29$, or $m = 50.75$. Therefore, the short-run total cost for producing 175 units (at a status quo of 17.5 units of labor) is

$$\$10 \times 17.5 + \$2.50 \times 50.75 = \$301.875.$$

Solution to Problem III.6

(a) Figure III.4 shows an iso-cost at these prices for total cost of \$50, slid parallel to find the minimum cost way to make 20 units: 60 units of labor and 40 units of material, for a total cost of \$100.¹

(b) Since this production technology has constant returns to scale, the 40-unit isoquant is shaped exactly like the 20-unit isoquant, except that it is twice as far out from the origin. Hence, the cost-minimizing way to produce 40 units is with 120 units of labor and 80 units of material, for a total cost of \$200.

(c) From part a (or b) and the fact of constant returns to scale, the firm has

¹ To answer the parenthetical question posed in this problem, if we allowed for convex combinations of the two technologies, the 20-unit isoquant would just be the convex hull of the 20-unit isoquant depicted; that is, the line joining the two corners would be added. Then, the least-cost iso-cost line hitting the isoquant would hit at one of the two "corners" first. If the iso-cost lines are parallel to the line segment joining the two corners, then the least-cost iso-cost hits the entire line segment all at once; in that case, either corner is as cheap as any convex combination of the two. So, as long as the firm can switch between the two technologies freely, allowing convex combinations does not reduce costs. Note: There are two complications of this simple story, either of which could make convex combinations useful—(1) if the separate technologies had rising marginal costs (decreasing returns to scale), or (2) if the firm had an effect on the cost of its inputs (that is, if the cost of an input rose the more the firm bought), so that iso-cost *lines* would become iso-cost *curves*.

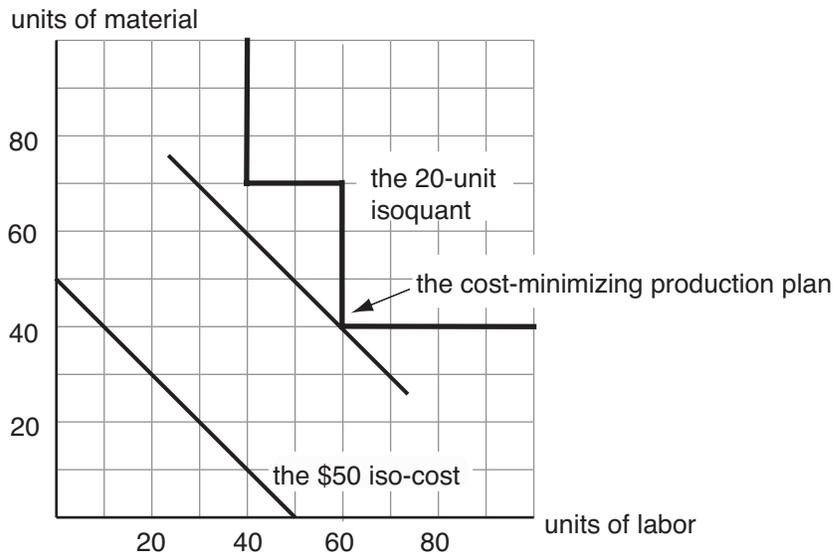


Figure III.4. Problem III.6: Finding the cost-minimizing way to produce 20 units.

constant average and marginal costs of \$5 per unit. Marginal revenue is

$$MR(x) = 25 - \frac{2x}{100}$$

so marginal cost equals marginal revenue where

$$25 - \frac{2x}{100} = 5 \quad \text{or} \quad 20 = \frac{2x}{100} \quad \text{or} \quad x = 1000.$$

Note that this entails using the 3 units of labor to 2 of material technology, with 3000 units of labor and 2000 units of material. The price of the good is $25 - (1000/100) = \$15$, so the profit margin on each unit is \$10 and the total profit is \$10,000.

(d) In the short run, the firm must continue to produce with fixed coefficients of 3 units of labor to 2 units of material per unit output. At the new input prices, this means that the marginal cost of each unit of output is

$$\$2 \times 3 + \$1 \times 2 = \$8.$$

So marginal cost equals marginal revenue where

$$25 - \frac{2x}{100} = 8 \quad \text{or} \quad 17 = \frac{2x}{100} \quad \text{or} \quad x = 850.$$

This gives a price for the good of $25 - (850/100) = \$16.50$, for a profit margin of \$8.50 per unit and a total profit of \$7225.

(e) In the longer run, at these prices, it makes sense for the manufacturer to switch to the 2 labor to 3.5 materials technology (draw the iso-cost if you do not see this), which gives a constant marginal cost of $\$2 \times 2 + \$1 \times 3.5 = \$7.50$ per unit. The firm maximizes profit where

$$25 - \frac{2x}{100} = 7.50 \quad \text{or} \quad 17.5 = \frac{2x}{100} \quad \text{or} \quad x = 875.$$

This gives a price of $25 - (875/100) = \$16.25$ per unit, for a profit margin of \$8.75 per unit and total profit of \$7656.25.

Solution to Problem III.7

(a) Labor costs for producing x trewqs are $\$10 \times 5 \times x = \$50x$. Material costs are $\$2 \times (2x + x^2/10) = \$4x + (x^2/5)$. So total costs are

$$TC(x) = \$54x + \frac{x^2}{5}.$$

(b) This inverse demand function gives a marginal revenue function of $MR(x) = 198 - 0.8x$. Marginal cost is $MC(x) = 54 + 0.4x$. So marginal cost equals marginal revenue where

$$198 - 0.8x = 54 + 0.4x \quad \text{or} \quad 144 = 1.2x \quad \text{or} \quad x = 120.$$

This gives a price of

$$198 - 0.4 \times 120 = \$150.$$

This implies the use of 600 units of labor and $240 + 1440 = 1680$ units of material.

(c) This acts as an increase in the marginal cost of a trewq of \$4, or an increase in total costs of $\$4x$ when you produce x trewqs. That is, the (long-run) total cost function becomes $TC(x) = 58x + x^2/5$ and the (long-run) marginal cost function becomes $58 + 2x/5$.

(d) If the firm wishes to produce precisely 120 trewqs, it would employ the same amount of labor as before, so that its (short-run) total costs would be

$58x + x^2/5$ at $x = 120$. If it wishes to reduce the amount of trewqs it produces, it must continue to employ all 600 labor hours, but it can reduce its materials to the appropriate level of $2x + x^2/10$, so its total costs would be

$$\$10 \times 600 + \$2 \times [2x + x^2/10] + \$4 \times x = 6000 + 8x + x^2/5.$$

If it wishes to increase trewq production beyond 120 trewqs, it must increase the amount of labor it hires by $5(x - 120)$ at a cost of \$15 per unit, for a total cost of

$$\$10 \times 600 + \$15 \times (5[x - 120]) + \$2 \times [2x + x^2/10] + \$4 \times x =$$

$$75x - 3000 + 8x + 2x^2/10 = 83x + 2x^2/10 - 3000.$$

In this last string of equalities, the leftmost expression is the key: This is the sum of the wages paid at regular time (for the “fixed” 600 hours), plus the wages paid for overtime work, plus material costs, plus the cost of the tax. (The rest is algebra.)

Therefore, the firm has a discontinuous short-run marginal cost function. For levels of production x less than 120 units, the short-run marginal costs are

$$\text{SRMC}(x) = 8 + 0.4x.$$

For levels of x greater than 120 units, the short-run marginal costs are

$$\text{SRMC}(x) = 83 + 0.4x.$$

This discontinuity in short-run marginal costs occurs because, on the margin, adding production beyond 120 units involves overtime labor, while subtracting production below 120 units gives no labor cost relief.

Figure III.5 graphs the short-run marginal cost and marginal revenue functions. Note that marginal revenue exceeds short-run marginal cost for all levels x below 120 and marginal cost exceeds marginal cost for all levels x

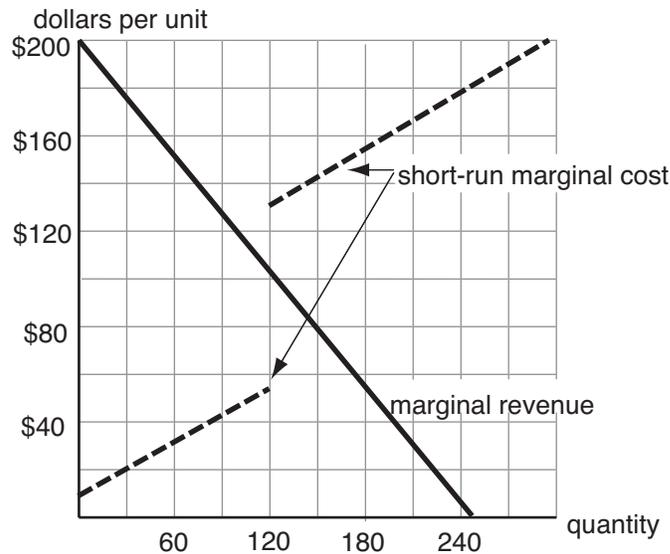


Figure III.5. Problem III.7: Short-run marginal cost and marginal revenue.

above 120, so the profit-maximizing level of production remains at 120 units. All that changes is that the firm pays \$480 in taxes to the government.

(e) In the long run, the firm's total cost function becomes $58x + x^2/5$ at all levels of x . So, in the long run, marginal cost equals marginal revenue where

$$58 + 0.4x = 198 - 0.8x \quad \text{or} \quad 140 = 1.2x \quad \text{or} \quad x = 116.667.$$

This leads to a price of \$151.33.

(f) Because the new technology has constant returns to scale, it has constant marginal = average cost. This marginal = average cost is

$$\$10 \times 4 + \$2 \times 6 + \$140 \times 0.1 = \$66.$$

Making 120 trewqs by this technology therefore costs $\$66 \times 120 = \7920 . (Compare this with a total cost for 120 trewqs of \$9360 using the old technology.)

(g) With the new technology (exclusively), marginal cost is \$66, so marginal cost equals marginal revenue at

$$198 - 0.8x = 66 \quad \text{or} \quad x = \frac{132}{0.8} = 165 \text{ units,}$$

at which point the price is $198 - (165)(0.4) = \$132$. This gives a profit of \$10,890. (Compare this with a profit of \$8640 with the old technology.)

(h) If the firm could “mix” the two technologies, it would use the original technology up to the level of production where its marginal cost rises to \$66, then use the second technology for all the rest. The joint marginal cost function (the horizontal sum of the two) traces out the first MC curve up to \$66, then it is flat at \$66.

To find the quantity at which the first technology has an MC of \$66, we solve

$$54 + 0.4x = 66 \quad \text{or} \quad 0.4x = 12 \quad \text{or} \quad x = 30.$$

To find the profit-maximizing level of production, we note that, beyond 30 units, MC is the same as in part g. Since the intersection of MC and MR in part g was well beyond 30 units, the intersection in this case is in the same position (see Figure III.6). Therefore, the profit-maximizing level of production is 165 units, sold at a price of \$132 apiece. The revenue side of the firm is the same. On the cost side, the firm saves a bit on the 30 units it makes with the old technology, relative to part g. Specifically, those 30 units cost $(54)(30) + 30^2/5 = \$1800$ if made with the old technology and $(66)(30) = \$1980$ if made with the new technology. This represents a savings (relative to part g) of \$180, increasing profit to \$11,070.

Solution to Problem III.8

In the short run, the firm is stuck with $l = 64$, so its short-run production function is $x = 64^{1/6}m^{1/3} = 2m^{1/3}$. Hence, in the short run, to produce x units (with $l = 64$) requires $x^3/8$ units of m , and the new short-run total-cost function (at the new price of m) is

$$\text{SRTC}(x) = \$300 + \$4 \times 64 + \$2 \times \frac{x^3}{8} = 556 + \frac{x^3}{4}.$$

Hence the short-run marginal-cost function is $3x^2/4$, and $\text{MR} = \text{MC}$ (in the short run) is

$$160 - 4x = \frac{3x^2}{4}.$$

Using the quadratic formula, this has solution $x = 12.181$. The corresponding level of m is 225.92 units.

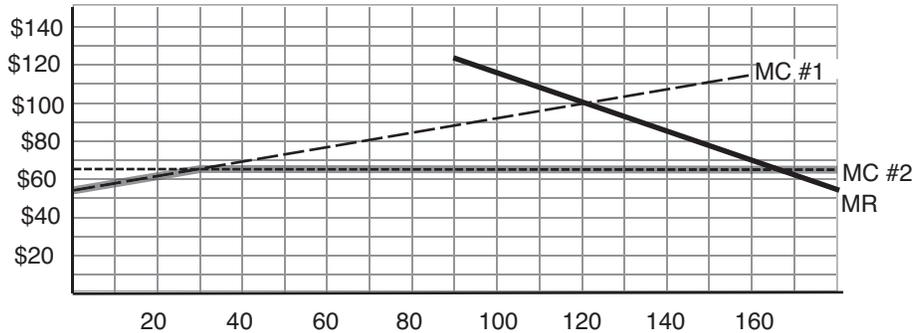


Figure III.6 Problem III.7(b), (g), and (h). In this figure, dashed lines represent the marginal cost functions for the original technology (large dashes) and the new technology (small dashes), labeled MC#1 and MC#2, respectively. Their horizontal sum, the marginal cost function of the “mix and match” technology (depicted in gray) follows the heavy dashes up to the quantity 30 and cost of \$66, and then is flat at \$66 for all quantities beyond 30. That is, with the ability to mix the two technologies, the firm produces the first 30 units with technology #1 and any other units with technology #2. The marginal revenue function is depicted by the downward-sloping solid line. To check your understanding, see if on this graph you can find the \$180 that the firm saves in part h (relative to part g) by using both technologies. (Hint: It is an area.)

For the long run, we must first find the new long-run total-cost function. With the new price of material of \$2, the cost-minimizing ratio of m to l is given by

$$2 \times 3 \times m = 4 \times 6 \times l \quad \text{or} \quad m = 4l.$$

Since $x = l^{1/6}m^{1/3}$, this gives $x = l^{1/6}(4l)^{1/3} = 1.587l^{1/2}$, so $l = 0.3969x^2$, and $m = 4l = 1.5874x^2$. Hence the new long-run total-cost function is

$$\$300 + \$4 \times 0.3969x^2 + \$2 \times 1.5874x^2 = 300 + 4.7622x^2.$$

And $MR = MC$ in the long run is

$$160 - 4x = 9.5244x \quad \text{or} \quad x = 11.8305.$$

This calls for 55.543 units of l and 222.172 units of m .

Solution to Problem III.9

(a) Marginal cost at the first facility is $5 + x_1/1000$, so the firm can produce 10,000 units at that facility before its marginal cost rises above \$15. And marginal cost at the second facility is $8 + x_2/2000$, so the firm can produce 14,000 units there before the marginal cost goes above \$15. Therefore, the firm would produce up to 24,000 units domestically before going to the foreign supplier, but any quantity beyond 24,000 would be sourced from the foreign supplier.

(b) Let $X_1^*(c)$ be the number of units that can be made at the first facility at marginal cost c or less. This is the solution to $c = 5 + x_1/1000$ or $X_1^*(c) = 1000(c - 5)$. Similarly, $X_2^*(c) = 2000(c - 8)$. Thus the total number (up to 24,000, at a marginal cost of \$15 or less) the firm can procure for marginal cost c or less is

$$X^*(c) = \begin{cases} 0, & \text{if } c < 5, \\ 1000(c - 5), & \text{if } 5 \leq c < 8, \text{ and} \\ 1000(c - 5) + 2000(c - 8) = 3000c - 21,000, & \text{if } c \geq 8. \end{cases}$$

Let me reiterate that this is true only up to $c = \$15$; at that point, the firm can get as many as it wants, at a \$15 marginal cost. The firm wishes to procure 15,000—below the 24,000 ceiling we computed in part a—so we solve $X^*(c) = 15,000$ for c . This gives $c = 12$. And thus the firm should source $X_1^*(12) = 7000$ from the first facility and $X_2^*(12) = 8000$ from the second.

(c) Marginal revenue is $MR(x) = 24 - x/3000$. This reaches \$15 when $x = 27,000$, which is beyond the point (24,000) at which marginal cost has risen to \$15. So the firm optimally produces 24,000 units domestically and procures another 3000 from the foreign supplier. The price will be \$19.50, and the firm will source 10,000 units from facility #1 and 14,000 from facility #2.